Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/7.1.2-d-x-^m-a+b-arcsinh-c-x-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [156]. This is test number [186].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (156)	% 0.00 (0)
Mathematica	% 100.00 (156)	% 0.00 (0)
Maple	% 69.87 (109)	% 30.13 (47)
Maxima	% 32.69 (51)	% 67.31 (105)
Fricas	% 27.56 (43)	% 72.44 (113)
Sympy	% 30.77 (48)	% 69.23 (108)
Giac	% 23.08 (36)	% 76.92 (120)
Mupad	% 19.23 (30)	% 80.77 (126)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

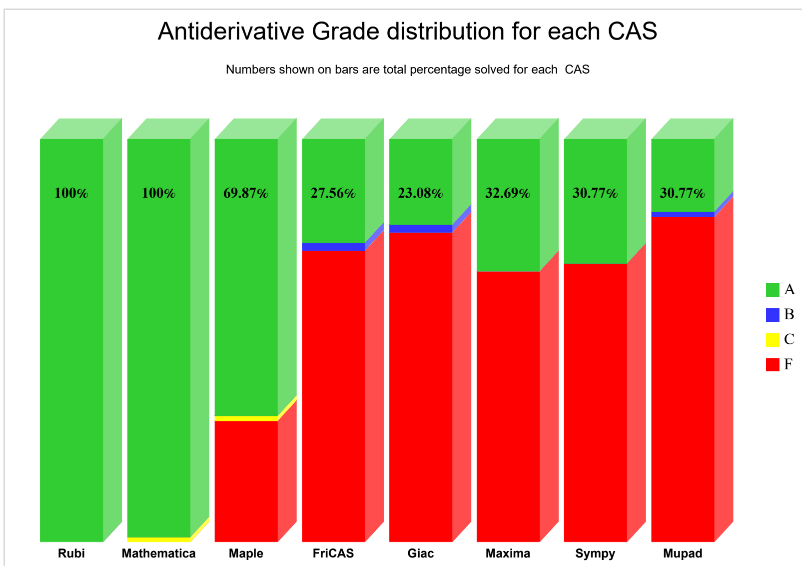
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

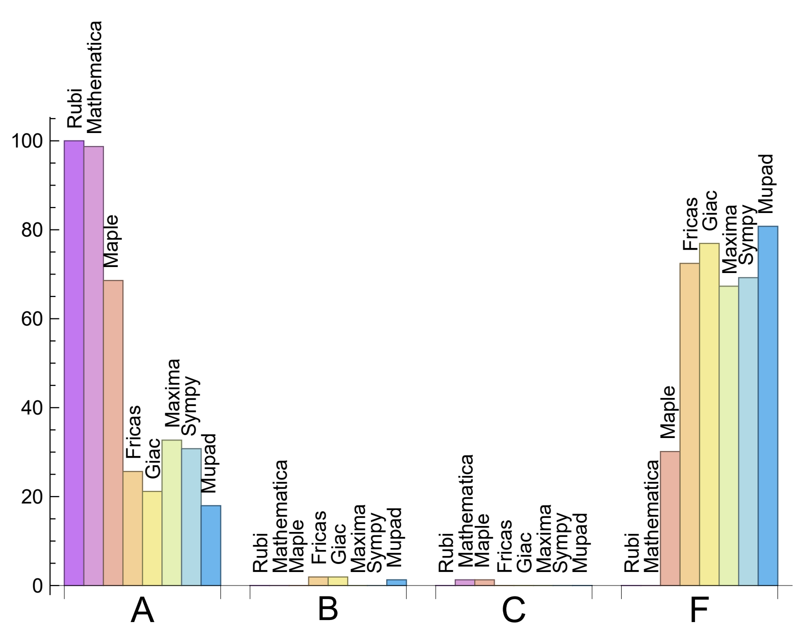
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	98.72	0.00	1.28	0.00
Maple	68.59	0.00	1.28	30.13
Maxima	32.69	0.00	0.00	67.31
Fricas	25.64	1.92	0.00	72.44
Sympy	30.77	0.00	0.00	69.23
Giac	21.15	1.92	0.00	76.92
Mupad	17.95	1.28	0.00	80.77

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	47	34.04 %	0.00 %	65.96 %
Maxima	105	100.00 %	0.00 %	0.00 %
Fricas	113	38.94 %	0.00 %	61.06 %
Sympy	108	98.15 %	1.85 %	0.00 %
Giac	120	62.50 %	3.33 %	34.17 %
Mupad	126	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

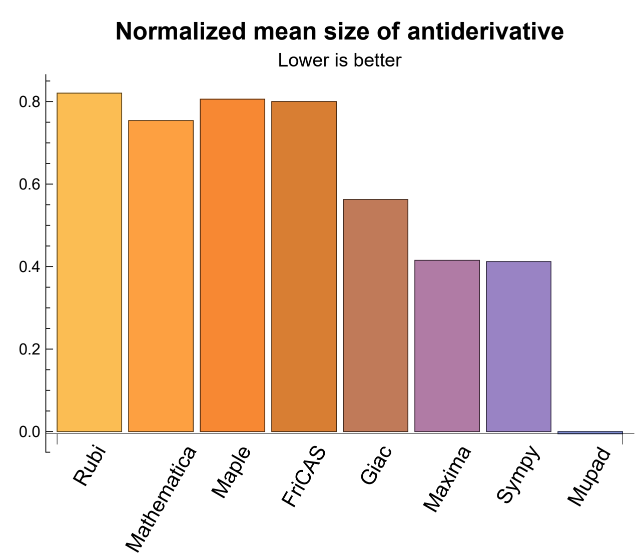
1.3 Performance

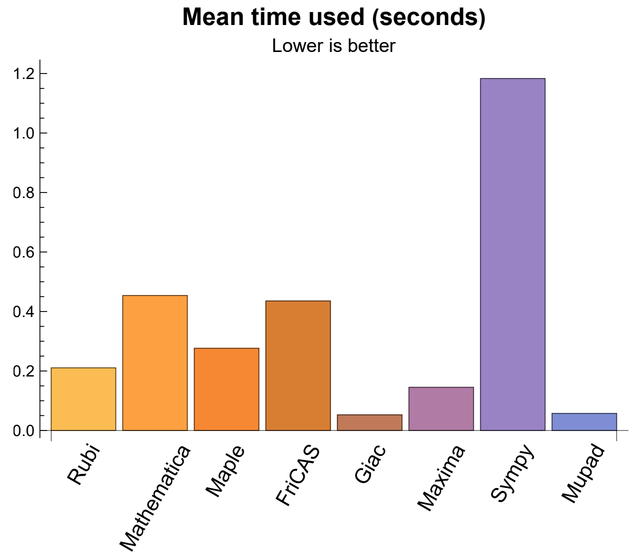
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	101.14	0.82	84.50	1.00
Mathematica	0.45	91.44	0.75	75.00	0.85
Maple	0.28	71.03	0.81	56.00	0.88
Maxima	0.15	33.31	0.42	0.00	0.00
Fricas	0.44	65.84	0.80	59.00	0.88
Sympy	1.18	47.00	0.41	0.00	0.00
Giac	0.05	28.00	0.56	0.00	0.00
Mupad	0.06	1.03	-0.01	-1.00	-0.07

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {18, 20, 28, 29, 30, 31, 39, 41, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

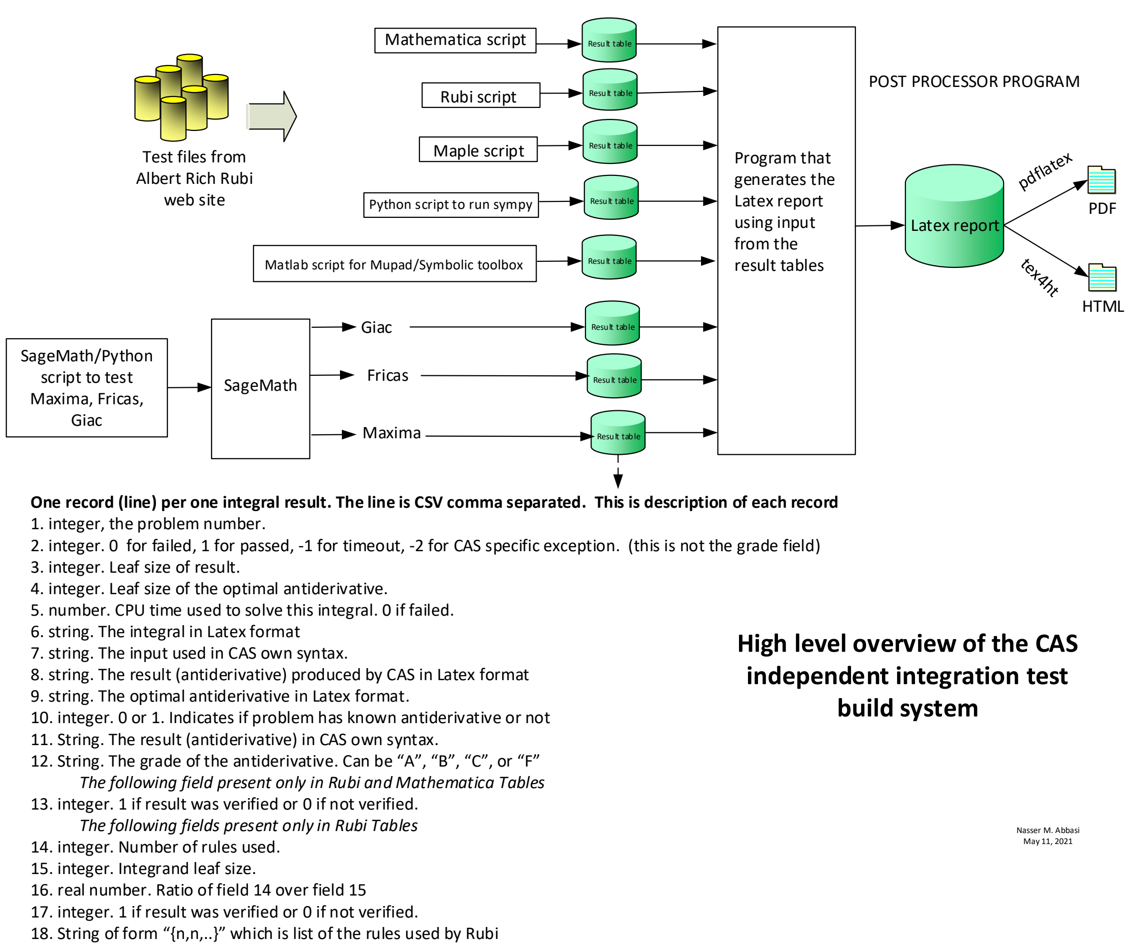
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade: { }

C grade: { 11, 40 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 78, 79, 83, 84, 85, 89, 90, 91, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 114, 115, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135 }

B grade: { }

C grade: { 132, 133 }

F grade: { 74, 75, 76, 80, 81, 82, 86, 87, 88, 92, 93, 94, 99, 100, 101, 105, 106, 107, 111, 112, 113, 119, 120, 129, 130, 131, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 24, 26, 33, 35, 37, 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135 }

B grade: { }

C grade: { }

F grade: { 6, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 10, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 58, 59, 65, 66, 72, 73, 117, 118, 121, 122, 128, 134, 135 }

B grade: { 7, 9, 11 }

C grade: { }

F grade: { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 117, 118, 121, 122, 124, 125, 126, 127, 128, 134, 135 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 123, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.7 Giac

A grade: { 4, 5, 8, 9, 10, 11, 16, 26, 37, 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 117, 118, 121, 122, 126, 127, 134, 135 }

B grade: { 7, 19, 21 }

C grade: { }

F grade: { 1, 2, 3, 6, 12, 13, 14, 15, 17, 18, 20, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 123, 124, 125, 128, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.8 Mupad

A grade: { 49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135 }

B grade: { 4, 5 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	50	69	68	61	70	0	-1
normalized size	1	1.00	0.69	0.96	0.94	0.85	0.97	0.00	-0.01
time (sec)	N/A	0.044	0.036	0.025	0.320	0.419	1.767	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	49	58	59	59	61	0	-1
normalized size	1	1.00	0.73	0.87	0.88	0.88	0.91	0.00	-0.01
time (sec)	N/A	0.026	0.017	0.021	0.320	0.417	0.882	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	50	48	52	48	0	-1
normalized size	1	1.00	0.79	0.96	0.92	1.00	0.92	0.00	-0.02
time (sec)	N/A	0.035	0.024	0.021	0.343	0.408	0.442	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	40	39	39	48	37	68	36
normalized size	1	1.00	0.91	0.89	0.89	1.09	0.84	1.55	0.82
time (sec)	N/A	0.016	0.011	0.020	0.311	0.434	0.196	0.141	1.653

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	37	20	35	23
normalized size	1	1.00	1.00	1.04	1.00	1.48	0.80	1.40	0.92
time (sec)	N/A	0.008	0.008	0.018	0.294	0.415	0.131	0.114	0.070

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	94	0	0	0	0	-1
normalized size	1	1.00	1.00	2.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.003	0.171	0.000	0.413	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	30	22	90	0	56	-1
normalized size	1	1.00	1.00	1.11	0.81	3.33	0.00	2.07	-0.04
time (sec)	N/A	0.021	0.002	0.022	0.316	0.467	0.000	0.128	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	28	37	27	36	0	50	-1
normalized size	1	1.00	0.85	1.12	0.82	1.09	0.00	1.52	-0.03
time (sec)	N/A	0.014	0.008	0.024	0.308	0.422	0.000	0.144	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	43	117	0	84	-1
normalized size	1	1.00	1.00	0.94	0.80	2.17	0.00	1.56	-0.02
time (sec)	N/A	0.032	0.010	0.020	0.333	0.437	0.000	0.124	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	40	56	49	49	0	77	-1
normalized size	1	1.00	0.71	1.00	0.88	0.88	0.00	1.38	-0.02
time (sec)	N/A	0.020	0.013	0.022	0.309	0.421	0.000	0.150	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	49	70	63	129	0	107	-1
normalized size	1	1.00	0.64	0.91	0.82	1.68	0.00	1.39	-0.01
time (sec)	N/A	0.045	0.013	0.029	0.297	0.445	0.000	0.125	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	75	103	99	99	114	0	-1
normalized size	1	1.00	0.64	0.88	0.85	0.85	0.97	0.00	-0.01
time (sec)	N/A	0.188	0.068	0.382	0.335	0.421	3.029	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	72	87	109	92	90	0	-1
normalized size	1	1.00	0.75	0.91	1.14	0.96	0.94	0.00	-0.01
time (sec)	N/A	0.165	0.047	0.376	0.331	0.409	1.859	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	59	72	70	82	76	0	-1
normalized size	1	1.00	0.74	0.90	0.88	1.02	0.95	0.00	-0.01
time (sec)	N/A	0.123	0.063	0.375	0.328	0.418	0.864	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	59	81	73	51	0	-1
normalized size	1	1.00	0.90	1.00	1.37	1.24	0.86	0.00	-0.02
time (sec)	N/A	0.092	0.032	0.079	0.341	0.413	0.434	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	32	59	32	62	-1
normalized size	1	1.00	1.00	1.06	0.94	1.74	0.94	1.82	-0.03
time (sec)	N/A	0.045	0.015	0.093	0.321	0.411	0.186	0.132	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	151	0	0	0	0	-1
normalized size	1	1.00	1.00	2.52	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.006	0.171	0.000	0.406	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	75	107	0	0	0	0	-1
normalized size	1	1.00	1.50	2.14	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.100	0.213	0.237	0.000	0.463	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	67	39	67	0	98	-1
normalized size	1	1.00	1.00	1.56	0.91	1.56	0.00	2.28	-0.02
time (sec)	N/A	0.080	0.034	0.360	0.307	0.416	0.000	0.171	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	125	144	0	0	0	0	-1
normalized size	1	1.00	1.26	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.531	0.478	0.000	0.421	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	99	71	85	0	148	-1
normalized size	1	1.00	0.75	1.16	0.84	1.00	0.00	1.74	-0.01
time (sec)	N/A	0.138	0.065	0.451	0.313	0.427	0.000	0.251	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	120	172	165	151	196	0	-1
normalized size	1	1.00	0.62	0.88	0.85	0.77	1.01	0.00	-0.01
time (sec)	N/A	0.366	0.076	0.484	0.346	0.411	5.446	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	110	141	0	142	160	0	-1
normalized size	1	1.00	0.67	0.87	0.00	0.87	0.98	0.00	-0.01
time (sec)	N/A	0.298	0.070	0.382	0.000	0.446	3.154	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	93	116	116	124	128	0	-1
normalized size	1	1.00	0.70	0.88	0.88	0.94	0.97	0.00	-0.01
time (sec)	N/A	0.221	0.060	0.379	0.330	0.420	1.802	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	80	88	0	112	92	0	-1
normalized size	1	1.00	0.82	0.91	0.00	1.15	0.95	0.00	-0.01
time (sec)	N/A	0.153	0.048	0.081	0.000	0.407	0.872	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	55	57	90	54	98	-1
normalized size	1	1.00	1.00	0.95	0.98	1.55	0.93	1.69	-0.02
time (sec)	N/A	0.081	0.017	0.099	0.311	0.398	0.427	0.176	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	204	0	0	0	0	-1
normalized size	1	1.00	1.00	2.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.007	0.161	0.000	0.413	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	117	162	0	0	0	0	-1
normalized size	1	1.00	1.39	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.127	0.231	0.000	0.417	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	149	0	0	0	0	-1
normalized size	1	1.00	0.86	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.345	0.366	0.000	0.464	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	268	228	0	0	0	0	-1
normalized size	1	1.00	1.77	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	2.250	0.540	0.000	0.427	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	107	210	0	0	0	0	-1
normalized size	1	1.00	0.67	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.286	0.691	0.467	0.000	0.428	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	165	242	0	208	269	0	-1
normalized size	1	1.00	0.60	0.88	0.00	0.75	0.97	0.00	-0.00
time (sec)	N/A	0.856	0.106	0.482	0.000	0.412	15.202	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	148	210	201	189	241	0	-1
normalized size	1	1.00	0.61	0.86	0.82	0.77	0.99	0.00	-0.00
time (sec)	N/A	0.657	0.091	0.480	0.346	0.425	8.860	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	133	172	0	176	190	0	-1
normalized size	1	1.00	0.69	0.89	0.00	0.91	0.98	0.00	-0.01
time (sec)	N/A	0.504	0.075	0.385	0.000	0.440	5.557	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	112	140	143	154	158	0	-1
normalized size	1	1.00	0.69	0.86	0.88	0.95	0.98	0.00	-0.01
time (sec)	N/A	0.363	0.075	0.380	0.335	0.425	3.079	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	94	105	0	138	104	0	-1
normalized size	1	1.00	0.85	0.95	0.00	1.25	0.95	0.00	-0.01
time (sec)	N/A	0.239	0.048	0.095	0.000	0.414	1.779	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	65	73	112	65	125	-1
normalized size	1	1.00	1.00	0.97	1.09	1.67	0.97	1.87	-0.01
time (sec)	N/A	0.125	0.020	0.095	0.308	0.407	0.829	0.239	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	257	0	0	0	0	-1
normalized size	1	1.00	1.00	2.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.009	0.159	0.000	0.418	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	161	217	0	0	0	0	-1
normalized size	1	1.00	1.34	1.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.247	0.235	0.000	0.428	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	113	208	0	0	0	0	-1
normalized size	1	1.00	1.05	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	0.281	0.369	0.000	0.420	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	355	372	0	0	0	0	-1
normalized size	1	1.00	1.59	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	2.641	0.485	0.000	0.432	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	0	0	0	-1
normalized size	1	1.00	0.73	0.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.100	0.013	0.195	0.000	0.414	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	0	0	0	-1
normalized size	1	1.00	0.77	0.77	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.083	0.113	0.191	0.000	0.441	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	0	0	0	-1
normalized size	1	1.00	0.76	0.76	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	0.010	0.164	0.000	0.402	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	0	0	0	-1
normalized size	1	1.00	0.83	0.83	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	0.077	0.157	0.000	0.402	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	0	0	0	0	-1
normalized size	1	1.00	0.81	0.81	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	0.007	0.127	0.000	0.416	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	0	-1
normalized size	1	1.00	1.00	0.93	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.038	0.022	0.161	0.000	0.396	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	-1
normalized size	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.018	0.009	0.100	0.000	0.398	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.014	0.190	0.265	0.000	0.431	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.015	0.869	0.364	0.000	0.392	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	85	104	0	0	0	0	-1
normalized size	1	1.00	1.04	1.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.295	0.208	0.000	0.399	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	78	78	0	0	0	0	-1
normalized size	1	1.00	1.11	1.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.042	0.196	0.000	0.397	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	80	0	0	0	0	-1
normalized size	1	1.00	0.88	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.213	0.171	0.000	0.386	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	54	0	0	0	0	-1
normalized size	1	1.00	1.00	0.96	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.028	0.155	0.000	0.388	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	56	0	0	0	0	-1
normalized size	1	1.00	0.91	1.04	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.174	0.137	0.000	0.394	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	28	0	0	0	0	-1
normalized size	1	1.00	0.86	0.76	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.004	0.165	0.000	0.452	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	30	0	0	0	0	-1
normalized size	1	1.00	0.91	0.88	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.083	0.051	0.093	0.000	0.403	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.013	0.786	0.245	0.000	0.409	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.014	5.059	0.355	0.000	0.430	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	102	120	0	0	0	0	-1
normalized size	1	1.00	1.05	1.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.157	0.168	0.000	0.472	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	82	0	0	0	0	-1
normalized size	1	1.00	0.84	1.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.221	0.156	0.000	0.468	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	81	0	0	0	0	-1
normalized size	1	1.00	0.79	1.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.160	0.138	0.000	0.436	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	43	0	0	0	0	-1
normalized size	1	1.00	0.98	0.68	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.171	0.053	0.168	0.000	0.459	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	42	0	0	0	0	-1
normalized size	1	1.00	0.94	0.84	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.083	0.022	0.092	0.000	0.434	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.013	0.564	0.236	0.000	0.413	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.014	5.316	0.350	0.000	0.441	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	156	169	0	0	0	0	-1
normalized size	1	1.00	1.01	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.368	0.191	0.000	0.423	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	105	114	0	0	0	0	-1
normalized size	1	1.00	0.74	0.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.422	0.181	0.000	0.433	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	99	115	0	0	0	0	-1
normalized size	1	1.00	0.72	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.306	0.330	0.137	0.000	0.421	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	84	60	0	0	0	0	-1
normalized size	1	1.00	0.88	0.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.132	0.169	0.000	0.421	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	61	0	0	0	0	-1
normalized size	1	1.00	0.91	0.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.063	0.102	0.000	0.434	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.013	1.910	0.243	0.000	0.878	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.014	9.338	0.367	0.000	0.394	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	161	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.324	0.048	180.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	101	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.039	180.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	101	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.037	180.000	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	52	75	0	0	0	0	-1
normalized size	1	1.00	0.56	0.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.035	0.295	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	45	42	0	0	0	0	-1
normalized size	1	1.00	0.85	0.79	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.107	0.053	0.301	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	0.369	0.358	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	152	0	0	0	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	0.130	180.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	102	0	0	0	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.487	0.038	180.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	102	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.371	0.038	180.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	52	102	0	0	0	0	-1
normalized size	1	1.00	0.43	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.023	0.348	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	47	65	0	0	0	0	-1
normalized size	1	1.00	0.58	0.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.030	0.338	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.015	0.340	0.346	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	152	0	0	0	0	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.996	0.126	180.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	101	0	0	0	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	0.039	180.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	101	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.551	0.038	180.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	52	136	0	0	0	0	-1
normalized size	1	1.00	0.34	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.337	0.036	0.354	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	45	78	0	0	0	0	-1
normalized size	1	1.00	0.48	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.052	0.339	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	0.343	0.355	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	151	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.112	180.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	99	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.081	180.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	99	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.077	180.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	37	0	0	0	0	-1
normalized size	1	1.00	0.83	0.59	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.022	0.206	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	47	24	0	0	0	0	-1
normalized size	1	1.00	1.09	0.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.029	0.193	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	0.333	0.352	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.015	1.713	0.607	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	216	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.306	180.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	126	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.041	180.000	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	140	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.142	180.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	80	0	0	0	0	-1
normalized size	1	1.00	0.93	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.028	0.359	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	69	65	0	0	0	0	-1
normalized size	1	1.00	1.08	1.02	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.073	0.355	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	0.419	0.355	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	343	0	0	0	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.584	0.363	180.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	174	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.441	0.434	180.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	225	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.372	0.157	180.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	98	119	0	0	0	0	-1
normalized size	1	1.00	0.83	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.185	0.332	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	105	81	0	0	0	0	-1
normalized size	1	1.00	1.25	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.107	0.321	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	0.423	0.358	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	334	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	0.754	180.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	210	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	0.735	180.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	221	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.397	180.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	118	147	0	0	0	0	-1
normalized size	1	1.00	0.80	1.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.346	0.342	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	111	105	0	0	0	0	-1
normalized size	1	1.00	0.99	0.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.200	0.328	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	0.421	0.350	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.877	1.324	0.000	0.489	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.106	0.787	1.188	0.000	0.434	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	123	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.041	1.206	0.000	0.440	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.023	1.201	0.000	0.444	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.015	0.424	1.011	0.000	0.442	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.014	0.447	0.992	0.000	0.407	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.015	1.118	0.375	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	1.120	0.243	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.013	1.414	0.246	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.014	1.287	0.245	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.015	1.205	0.237	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.018	0.793	0.155	0.000	0.477	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	145	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.155	180.000	0.000	0.459	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.069	180.000	0.000	0.428	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.075	180.000	0.000	0.499	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	38	0	0	0	0	-1
normalized size	1	1.00	1.00	0.64	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	0.017	0.161	0.000	0.422	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	45	40	0	0	0	0	-1
normalized size	1	1.00	0.92	0.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.027	0.142	0.000	0.424	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.015	0.289	0.161	0.000	0.419	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.016	1.125	0.254	0.000	0.462	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	215	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.605	0.420	180.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	127	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.432	0.099	0.382	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.241	0.365	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	215	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	0.313	180.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	129	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.478	0.107	0.395	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	251	0	0	0	0	0	-1
normalized size	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	1.110	0.394	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	215	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.250	0.497	180.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	115	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.751	0.077	0.378	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	282	0	0	0	0	0	-1
normalized size	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.406	3.646	0.371	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	196	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.350	0.234	180.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	108	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.068	0.372	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	101	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.103	0.362	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	290	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.409	180.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	134	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.115	0.379	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.246	0.378	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	340	0	0	0	0	0	-1
normalized size	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.899	1.622	180.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	200	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.517	0.746	0.378	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	181	0	0	0	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.602	0.363	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	417	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.033	1.740	180.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	208	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.506	1.220	0.374	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	210	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.447	0.631	0.372	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [30] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	8	0.375
2	A	4	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375
4	A	3	3	1.00	6	0.500
5	A	2	2	1.00	4	0.500
6	A	5	5	1.00	8	0.625
7	A	4	4	1.00	8	0.500
8	A	2	2	1.00	8	0.250
9	A	5	5	1.00	8	0.625
10	A	3	3	1.00	8	0.375
11	A	6	5	1.00	8	0.625
12	A	7	5	1.00	10	0.500
13	A	6	4	1.00	10	0.400
14	A	5	5	1.00	10	0.500
15	A	4	4	1.00	8	0.500
16	A	3	3	1.00	6	0.500
17	A	6	6	1.00	10	0.600
18	A	7	5	1.00	10	0.500
19	A	3	3	1.00	10	0.300
20	A	9	7	1.00	10	0.700

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	5	5	1.00	10	0.500
22	A	14	7	1.00	10	0.700
23	A	11	5	1.00	10	0.500
24	A	9	7	1.00	10	0.700
25	A	6	5	1.00	8	0.625
26	A	4	3	1.00	6	0.500
27	A	7	7	1.00	10	0.700
28	A	9	6	1.00	10	0.600
29	A	7	7	1.00	10	0.700
30	A	14	10	1.00	10	1.000
31	A	10	9	1.00	10	0.900
32	A	23	4	1.00	10	0.400
33	A	19	6	1.00	10	0.600
34	A	14	4	1.00	10	0.400
35	A	11	6	1.00	10	0.600
36	A	7	4	1.00	8	0.500
37	A	5	3	1.00	6	0.500
38	A	8	7	1.00	10	0.700
39	A	11	7	1.00	10	0.700
40	A	8	8	1.00	10	0.800
41	A	19	10	1.00	10	1.000
42	A	7	3	1.00	10	0.300
43	A	6	3	1.00	10	0.300
44	A	6	3	1.00	10	0.300
45	A	5	3	1.00	10	0.300
46	A	5	3	1.00	10	0.300
47	A	4	4	1.00	8	0.500
48	A	2	2	1.00	6	0.333
49	A	0	0	0.00	0	0.000
50	A	0	0	0.00	0	0.000
51	A	6	2	1.00	10	0.200
52	A	5	2	1.00	10	0.200
53	A	5	2	1.00	10	0.200
54	A	4	2	1.00	10	0.200
55	A	4	2	1.00	10	0.200
56	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
57	A	3	3	1.00	6	0.500
58	A	0	0	0.00	0	0.000
59	A	0	0	0.00	0	0.000
60	A	14	5	1.00	10	0.500
61	A	12	6	1.00	10	0.600
62	A	10	6	1.00	10	0.600
63	A	7	7	1.00	8	0.875
64	A	4	4	1.00	6	0.667
65	A	0	0	0.00	0	0.000
66	A	0	0	0.00	0	0.000
67	A	12	4	1.00	10	0.400
68	A	9	4	1.00	10	0.400
69	A	10	6	1.00	10	0.600
70	A	5	5	1.00	8	0.625
71	A	5	4	1.00	6	0.667
72	A	0	0	0.00	0	0.000
73	A	0	0	0.00	0	0.000
74	A	19	7	1.00	12	0.583
75	A	14	7	1.00	12	0.583
76	A	14	7	1.00	12	0.583
77	A	9	7	1.00	10	0.700
78	A	7	6	1.00	8	0.750
79	A	0	0	0.00	0	0.000
80	A	41	10	1.00	12	0.833
81	A	25	10	1.00	12	0.833
82	A	22	10	1.00	12	0.833
83	A	11	10	1.00	10	1.000
84	A	8	7	1.00	8	0.875
85	A	0	0	0.00	0	0.000
86	A	44	10	1.00	12	0.833
87	A	27	9	1.00	12	0.750
88	A	24	10	1.00	12	0.833
89	A	12	9	1.00	10	0.900
90	A	9	7	1.00	8	0.875
91	A	0	0	0.00	0	0.000
92	A	18	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	13	6	1.00	12	0.500
94	A	13	6	1.00	12	0.500
95	A	8	7	1.00	10	0.700
96	A	6	5	1.00	8	0.625
97	A	0	0	0.00	0	0.000
98	A	0	0	0.00	0	0.000
99	A	17	5	1.00	12	0.417
100	A	12	5	1.00	12	0.417
101	A	12	5	1.00	12	0.417
102	A	6	5	1.00	10	0.500
103	A	7	6	1.00	8	0.750
104	A	0	0	0.00	0	0.000
105	A	34	8	1.00	12	0.667
106	A	24	9	1.00	12	0.750
107	A	22	9	1.00	12	0.750
108	A	11	10	1.00	10	1.000
109	A	8	7	1.00	8	0.875
110	A	0	0	0.00	0	0.000
111	A	32	7	1.00	12	0.583
112	A	21	7	1.00	12	0.583
113	A	22	9	1.00	12	0.750
114	A	9	8	1.00	10	0.800
115	A	9	7	1.00	8	0.875
116	A	0	0	0.00	0	0.000
117	A	0	0	0.00	0	0.000
118	A	0	0	0.00	0	0.000
119	A	2	2	1.00	10	0.200
120	A	2	2	1.00	8	0.250
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	0	0	0.00	0	0.000
124	A	0	0	0.00	0	0.000
125	A	0	0	0.00	0	0.000
126	A	0	0	0.00	0	0.000
127	A	0	0	0.00	0	0.000
128	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	12	4	1.00	10	0.400
130	A	9	4	1.00	10	0.400
131	A	9	4	1.00	10	0.400
132	A	6	5	1.00	8	0.625
133	A	4	3	1.00	6	0.500
134	A	0	0	0.00	0	0.000
135	A	0	0	0.00	0	0.000
136	A	14	7	1.00	16	0.438
137	A	9	7	1.00	14	0.500
138	A	7	6	1.00	12	0.500
139	A	22	10	1.00	16	0.625
140	A	11	10	1.00	14	0.714
141	A	8	7	1.00	12	0.583
142	A	24	10	1.00	16	0.625
143	A	12	9	1.00	14	0.643
144	A	9	7	1.00	12	0.583
145	A	13	6	1.00	16	0.375
146	A	8	7	1.00	14	0.500
147	A	6	5	1.00	12	0.417
148	A	12	5	1.00	16	0.312
149	A	6	5	1.00	14	0.357
150	A	7	6	1.00	12	0.500
151	A	22	9	1.00	16	0.562
152	A	11	10	1.00	14	0.714
153	A	8	7	1.00	12	0.583
154	A	22	9	1.00	16	0.562
155	A	9	8	1.00	14	0.571
156	A	9	7	1.00	12	0.583

Chapter 3

Listing of integrals

3.1 $\int x^4 \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=72

$$-\frac{\left(a^2x^2+1\right)^{5/2}}{25a^5}+\frac{2\left(a^2x^2+1\right)^{3/2}}{15a^5}-\frac{\sqrt{a^2x^2+1}}{5a^5}+\frac{1}{5}x^5\sinh^{-1}(ax)$$

```
[Out] 2/15*(a^2*x^2+1)^(3/2)/a^5-1/25*(a^2*x^2+1)^(5/2)/a^5+1/5*x^5*arcsinh(a*x)-
1/5*(a^2*x^2+1)^(1/2)/a^5
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5661, 266, 43}

$$-\frac{\left(a^2x^2+1\right)^{5/2}}{25a^5}+\frac{2\left(a^2x^2+1\right)^{3/2}}{15a^5}-\frac{\sqrt{a^2x^2+1}}{5a^5}+\frac{1}{5}x^5\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x],x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(5*a^5) + (2*(1 + a^2*x^2)^(3/2))/(15*a^5) - (1 + a^2*x^
2)^(5/2)/(25*a^5) + (x^5*ArcSinh[a*x])/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \sinh^{-1}(ax)\,dx &= \frac{1}{5}x^5\sinh^{-1}(ax) - \frac{1}{5}a\int \frac{x^5}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{1}{5}x^5\sinh^{-1}(ax) - \frac{1}{10}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5\sinh^{-1}(ax) - \frac{1}{10}a\,\text{Subst}\left(\int \left(\frac{1}{a^4\sqrt{1+a^2x}} - \frac{2\sqrt{1+a^2x}}{a^4} + \frac{\left(1+a^2x\right)^{3/2}}{a^4}\right)dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{5a^5} + \frac{2\left(1+a^2x^2\right)^{3/2}}{15a^5} - \frac{\left(1+a^2x^2\right)^{5/2}}{25a^5} + \frac{1}{5}x^5\sinh^{-1}(ax)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.69

$$
\frac{1}{5}x^5\sinh^{-1}(ax) - \frac{\sqrt{a^2x^2+1}\left(3a^4x^4 - 4a^2x^2 + 8\right)}{75a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x],x]
```

```
[Out] -1/75*(Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 + 3*a^4*x^4))/a^5 + (x^5*ArcSinh[a*
x])/5
```

fricas [A] time = 0.42, size = 61, normalized size = 0.85

$$
\frac{15\,a^5x^5\log\left(ax + \sqrt{a^2x^2+1}\right) - \left(3\,a^4x^4 - 4\,a^2x^2 + 8\right)\sqrt{a^2x^2+1}}{75\,a^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1)) - (3*a^4*x^4 - 4*a^2*x^2 + 8)
*sqrt(a^2*x^2 + 1))/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 69, normalized size = 0.96

$$
\frac{\frac{a^5x^5\,\mathrm{arcsinh}(ax)}{5} - \frac{a^4x^4\sqrt{a^2x^2+1}}{25} + \frac{4a^2x^2\sqrt{a^2x^2+1}}{75} - \frac{8\sqrt{a^2x^2+1}}{75}}{a^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x),x)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)-1/25*a^4*x^4*(a^2*x^2+1)^(1/2)+4/75*a^2*x^2
*(a^2*x^2+1)^(1/2)-8/75*(a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.32, size = 68, normalized size = 0.94

$$\frac{1}{5}\,x^5\,\mathrm{arsinh}\left(ax\right)-\frac{1}{75}\left(\frac{3\,\sqrt{a^2x^2+1}\,x^4}{a^2}-\frac{4\,\sqrt{a^2x^2+1}\,x^2}{a^4}+\frac{8\,\sqrt{a^2x^2+1}}{a^6}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x) - 1/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2 +
 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{asinh}\left(a\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asinh(a*x),x)
```

```
[Out] int(x^4*asinh(a*x), x)
```

sympy [A] time = 1.77, size = 70, normalized size = 0.97

$$\begin{cases} \frac{x^5\,\mathrm{asinh}\left(ax\right)}{5}-\frac{x^4\sqrt{a^2x^2+1}}{25a}+\frac{4x^2\sqrt{a^2x^2+1}}{75a^3}-\frac{8\sqrt{a^2x^2+1}}{75a^5} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x),x)
```

```
[Out] Piecewise((x**5*asinh(a*x)/5 - x**4*sqrt(a**2*x**2 + 1)/(25*a) + 4*x**2*sqr
t(a**2*x**2 + 1)/(75*a**3) - 8*sqrt(a**2*x**2 + 1)/(75*a**5), Ne(a, 0)), (0
, True))
```

3.2 $\int x^3 \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=67

$$-\frac{3\sinh^{-1}(ax)}{32a^4}-\frac{x^3\sqrt{a^2x^2+1}}{16a}+\frac{3x\sqrt{a^2x^2+1}}{32a^3}+\frac{1}{4}x^4\sinh^{-1}(ax)$$

```
[Out] -3/32*arcsinh(a*x)/a^4+1/4*x^4*arcsinh(a*x)+3/32*x*(a^2*x^2+1)^(1/2)/a^3-1/
16*x^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5661, 321, 215}

$$-\frac{x^3\sqrt{a^2x^2+1}}{16a}+\frac{3x\sqrt{a^2x^2+1}}{32a^3}-\frac{3\sinh^{-1}(ax)}{32a^4}+\frac{1}{4}x^4\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x],x]
```

```
[Out] (3*x*Sqrt[1 + a^2*x^2])/(32*a^3) - (x^3*Sqrt[1 + a^2*x^2])/(16*a) - (3*ArcS
inh[a*x])/(32*a^4) + (x^4*ArcSinh[a*x])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\sinh^{-1}(ax)\,dx &= \frac{1}{4}x^4\sinh^{-1}(ax)-\frac{1}{4}a\int\frac{x^4}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{x^3\sqrt{1+a^2x^2}}{16a}+\frac{1}{4}x^4\sinh^{-1}(ax)+\frac{3\int\frac{x^2}{\sqrt{1+a^2x^2}}\,dx}{16a}\\
&= \frac{3x\sqrt{1+a^2x^2}}{32a^3}-\frac{x^3\sqrt{1+a^2x^2}}{16a}+\frac{1}{4}x^4\sinh^{-1}(ax)-\frac{3\int\frac{1}{\sqrt{1+a^2x^2}}\,dx}{32a^3}\\
&= \frac{3x\sqrt{1+a^2x^2}}{32a^3}-\frac{x^3\sqrt{1+a^2x^2}}{16a}-\frac{3\sinh^{-1}(ax)}{32a^4}+\frac{1}{4}x^4\sinh^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.73

$$\frac{(8a^4x^4-3)\sinh^{-1}(ax)+ax\sqrt{a^2x^2+1}\,(3-2a^2x^2)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x],x]
```

```
[Out] (a*x*(3 - 2*a^2*x^2)*Sqrt[1 + a^2*x^2] + (-3 + 8*a^4*x^4)*ArcSinh[a*x])/(32
*a^4)
```

fricas [A] time = 0.42, size = 59, normalized size = 0.88

$$\frac{(8\,a^4x^4-3)\log\left(ax+\sqrt{a^2x^2+1}\right)-(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}}{32\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1)) - (2*a^3*x^3 - 3*a*x)*sq
rt(a^2*x^2 + 1))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 58, normalized size = 0.87

$$\frac{\frac{a^4x^4\,\mathrm{arcsinh}(ax)}{4}-\frac{a^3x^3\sqrt{a^2x^2+1}}{16}+\frac{3ax\sqrt{a^2x^2+1}}{32}-\frac{3\,\mathrm{arcsinh}(ax)}{32}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x),x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)-1/16*a^3*x^3*(a^2*x^2+1)^(1/2)+3/32*a*x*(a^
2*x^2+1)^(1/2)-3/32*arcsinh(a*x))
```

maxima [A] time = 0.32, size = 59, normalized size = 0.88

$$\frac{1}{4}\,x^4\,\mathrm{arsinh}\,(ax)-\frac{1}{32}\left(\frac{2\,\sqrt{a^2x^2+1}\,x^3}{a^2}-\frac{3\,\sqrt{a^2x^2+1}\,x}{a^4}+\frac{3\,\mathrm{arsinh}\,(ax)}{a^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x) - 1/32*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 +
 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{asinh}\,(a\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x),x)
```

```
[Out] int(x^3*asinh(a*x), x)
```

sympy [A] time = 0.88, size = 61, normalized size = 0.91

$$\begin{cases} \frac{x^4 \operatorname{asinh}(ax)}{4} - \frac{x^3 \sqrt{a^2x^2+1}}{16a} + \frac{3x\sqrt{a^2x^2+1}}{32a^3} - \frac{3 \operatorname{asinh}(ax)}{32a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x),x)
```

```
[Out] Piecewise((x**4*asinh(a*x)/4 - x**3*sqrt(a**2*x**2 + 1)/(16*a) + 3*x*sqrt(a
**2*x**2 + 1)/(32*a**3) - 3*asinh(a*x)/(32*a**4), Ne(a, 0)), (0, True))
```

3.3 $\int x^2 \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=52

$$-\frac{(a^2x^2+1)^{3/2}}{9a^3}+\frac{\sqrt{a^2x^2+1}}{3a^3}+\frac{1}{3}x^3\sinh^{-1}(ax)$$

```
[Out] -1/9*(a^2*x^2+1)^(3/2)/a^3+1/3*x^3*arcsinh(a*x)+1/3*(a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5661, 266, 43}

$$-\frac{(a^2x^2+1)^{3/2}}{9a^3}+\frac{\sqrt{a^2x^2+1}}{3a^3}+\frac{1}{3}x^3\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x],x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(3*a^3) - (1 + a^2*x^2)^(3/2)/(9*a^3) + (x^3*ArcSinh[a*x]
)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\sinh^{-1}(ax)\,dx &= \frac{1}{3}x^3\sinh^{-1}(ax)-\frac{1}{3}a\int\frac{x^3}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{1}{3}x^3\sinh^{-1}(ax)-\frac{1}{6}a\,\text{Subst}\left(\int\frac{x}{\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\sinh^{-1}(ax)-\frac{1}{6}a\,\text{Subst}\left(\int\left(-\frac{1}{a^2\sqrt{1+a^2x}}+\frac{\sqrt{1+a^2x}}{a^2}\right)dx,x,x^2\right)\\
&= \frac{\sqrt{1+a^2x^2}}{3a^3}-\frac{(1+a^2x^2)^{3/2}}{9a^3}+\frac{1}{3}x^3\sinh^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.79

$$\frac{1}{9}\left(\frac{\left(2-a^2x^2\right)\sqrt{a^2x^2+1}}{a^3}+3x^3\sinh^{-1}(ax)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x],x]
```

```
[Out] (((2 - a^2*x^2)*Sqrt[1 + a^2*x^2])/a^3 + 3*x^3*ArcSinh[a*x])/9
```

fricas [A] time = 0.41, size = 52, normalized size = 1.00

$$\frac{3\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}\left(a^2x^2-2\right)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*(a^2*x^2 -
2))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 50, normalized size = 0.96

$$\frac{\frac{a^3x^3\,\mathrm{arcsinh}(ax)}{3}-\frac{a^2x^2\sqrt{a^2x^2+1}}{9}+\frac{2\sqrt{a^2x^2+1}}{9}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x),x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)-1/9*a^2*x^2*(a^2*x^2+1)^(1/2)+2/9*(a^2*x^2+
1)^(1/2))
```

maxima [A] time = 0.34, size = 48, normalized size = 0.92

$$\frac{1}{3}\,x^3\,\mathrm{arsinh}\,(ax)-\frac{1}{9}\,a\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2}-\frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x) - 1/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 +
1)/a^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\mathrm{asinh}\,(a\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x),x)
```

```
[Out] int(x^2*asinh(a*x), x)
```

sympy [A] time = 0.44, size = 48, normalized size = 0.92

$$\begin{cases} \frac{x^3 \operatorname{asinh}(ax)}{3} - \frac{x^2\sqrt{a^2x^2+1}}{9a} + \frac{2\sqrt{a^2x^2+1}}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x),x)
```

```
[Out] Piecewise((x**3*asinh(a*x)/3 - x**2*sqrt(a**2*x**2 + 1)/(9*a) + 2*sqrt(a**2
*x**2 + 1)/(9*a**3), Ne(a, 0)), (0, True))
```

3.4 $\int x \sinh^{-1}(ax)\, dx$

Optimal. Leaf size=44

$$-\frac{x\sqrt{a^2x^2+1}}{4a}+\frac{\sinh^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)$$

```
[Out] 1/4*arcsinh(a*x)/a^2+1/2*x^2*arcsinh(a*x)-1/4*x*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5661, 321, 215}

$$-\frac{x\sqrt{a^2x^2+1}}{4a}+\frac{\sinh^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x],x]
```

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(4*a) + ArcSinh[a*x]/(4*a^2) + (x^2*ArcSinh[a*x])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\sinh^{-1}(ax)\,dx &= \frac{1}{2}x^2\sinh^{-1}(ax)-\frac{1}{2}a\int\frac{x^2}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{x\sqrt{1+a^2x^2}}{4a}+\frac{1}{2}x^2\sinh^{-1}(ax)+\frac{\int\frac{1}{\sqrt{1+a^2x^2}}\,dx}{4a}\\
&= -\frac{x\sqrt{1+a^2x^2}}{4a}+\frac{\sinh^{-1}(ax)}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.91

$$\frac{\left(2a^2x^2+1\right)\sinh^{-1}(ax)-ax\sqrt{a^2x^2+1}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x],x]
```

```
[Out] (-(a*x*Sqrt[1 + a^2*x^2]) + (1 + 2*a^2*x^2)*ArcSinh[a*x])/(4*a^2)
```

fricas [A] time = 0.43, size = 48, normalized size = 1.09

$$-\frac{\sqrt{a^2x^2+1}\,ax-\left(2\,a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(a^2*x^2 + 1)*a*x - (2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))
/a^2
```

giac [A] time = 0.14, size = 68, normalized size = 1.55

$$\frac{1}{2}\,x^2\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2+1}\,x}{a^2}+\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{a^2|a|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1)) - 1/4*a*(sqrt(a^2*x^2 + 1)*x/a^2 + log
(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^2*abs(a)))
```

maple [A] time = 0.02, size = 39, normalized size = 0.89

$$\frac{\frac{a^2x^2\operatorname{arcsinh}(ax)}{2}-\frac{ax\sqrt{a^2x^2+1}}{4}+\frac{\operatorname{arcsinh}(ax)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x),x)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arcsinh(a*x)-1/4*a*x*(a^2*x^2+1)^(1/2)+1/4*arcsinh(a*x))
```

maxima [A] time = 0.31, size = 39, normalized size = 0.89

$$\frac{1}{2}\,x^2\operatorname{arsinh}(ax)-\frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2+1}\,x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a*x) - 1/4*a*(sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)
```

mupad [B] time = 1.65, size = 36, normalized size = 0.82

$$x\operatorname{asinh}(a\,x)\left(\frac{x}{2}+\frac{1}{4\,a^2\,x}\right)-\frac{x\sqrt{a^2\,x^2+1}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x),x)
```

```
[Out] x*asinh(a*x)*(x/2 + 1/(4*a^2*x)) - (x*(a^2*x^2 + 1)^(1/2))/(4*a)
```

sympy [**A**] time = 0.20, size = 37, normalized size = 0.84

$$\begin{cases} \frac{x^2 \operatorname{asinh}(ax)}{2} - \frac{x\sqrt{a^2x^2+1}}{4a} + \frac{\operatorname{asinh}(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x),x)
```

```
[Out] Piecewise((x**2*asinh(a*x)/2 - x*sqrt(a**2*x**2 + 1)/(4*a) + asinh(a*x)/(4*
a**2), Ne(a, 0)), (0, True))
```

3.5 $\int \sinh^{-1}(ax)\,dx$

Optimal. Leaf size=25

$$x\sinh^{-1}(ax) - \frac{\sqrt{a^2x^2+1}}{a}$$

```
[Out] x*arcsinh(a*x)-(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5653, 261}

$$x\sinh^{-1}(ax) - \frac{\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x],x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \sinh^{-1}(ax)\,dx &= x\sinh^{-1}(ax) - a\int \frac{x}{\sqrt{1+a^2x^2}}\,dx \\ &= -\frac{\sqrt{1+a^2x^2}}{a} + x\sinh^{-1}(ax) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$x\sinh^{-1}(ax) - \frac{\sqrt{a^2x^2+1}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x],x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

fricas [A] time = 0.42, size = 37, normalized size = 1.48

$$\frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right) - \sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x),x, algorithm="fricas")
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1))/a
```

giac [A]   time = 0.11, size = 35, normalized size = 1.40

$$x \log\left(ax + \sqrt{a^2x^2 + 1}\right) - \frac{\sqrt{a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x),x, algorithm="giac")
[Out] x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a
```

maple [A]   time = 0.02, size = 26, normalized size = 1.04

$$\frac{ax \operatorname{arcsinh}(ax) - \sqrt{a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x),x)
[Out] 1/a*(a*x*arcsinh(a*x)-(a^2*x^2+1)^(1/2))
```

maxima [A]   time = 0.29, size = 25, normalized size = 1.00

$$\frac{ax \operatorname{arsinh}(ax) - \sqrt{a^2x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x),x, algorithm="maxima")
[Out] (a*x*arcsinh(a*x) - sqrt(a^2*x^2 + 1))/a
```

mupad [B]   time = 0.07, size = 23, normalized size = 0.92

$$x \operatorname{asinh}(a\,x) - \frac{\sqrt{a^2\,x^2 + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x),x)
[Out] x*asinh(a*x) - (a^2*x^2 + 1)^(1/2)/a
```

sympy [A]   time = 0.13, size = 20, normalized size = 0.80

$$\begin{cases} x \operatorname{asinh}(ax) - \frac{\sqrt{a^2x^2+1}}{a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x),x)
[Out] Piecewise((x*asinh(a*x) - sqrt(a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))
```

3.6 $\int \frac{\sinh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\sinh^{-1}(ax)^2+\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

```
[Out] -1/2*arcsinh(a*x)^2+arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+1/2*polylo
g(2,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5659, 3716, 2190, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\sinh^{-1}(ax)^2+\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x,x]
```

```
[Out] -ArcSinh[a*x]^2/2 + ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + PolyLog[2, E
^(2*ArcSinh[a*x])]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x}\,dx = \text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{1}{2}\sinh^{-1}(ax)^2 - 2\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{1}{2}\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}(ax\right.$$
$$= -\frac{1}{2}\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}(ax)}\right.$$
$$= -\frac{1}{2}\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{1}{2}\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{1}{2}\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{1}{2}\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x,x]
```

```
[Out] -1/2*ArcSinh[a*x]^2 + ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + PolyLog[2,
 E^(2*ArcSinh[a*x])]/2
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/x, x)
```

maple [A] time = 0.17, size = 94, normalized size = 2.19

$$-\frac{\text{arcsinh}\,(ax)^2}{2} + \text{arcsinh}\,(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right) + \text{polylog}\left(2, ax+\sqrt{a^2x^2+1}\right) + \text{arcsinh}\,(ax)\ln\left(ax+\sqrt{a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x,x)
```

```
[Out] -1/2*arcsinh(a*x)^2+arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+polylog(2,a*x+
(a^2*x^2+1)^(1/2))+arcsinh(a*x)*ln(a*x+(a^2*x^2+1)^(1/2)+1)+polylog(2,-a*x-
(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/x,x)
```

```
[Out] int(asinh(a*x)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x,x)
```

```
[Out] Integral(asinh(a*x)/x, x)
```

3.7 $\int \frac{\sinh^{-1}(ax)}{x^2} \, dx$

Optimal. Leaf size=27

$$-a \tanh^{-1}\left(\sqrt{a^2x^2+1}\right) - \frac{\sinh^{-1}(ax)}{x}$$

```
[Out] -arcsinh(a*x)/x-a*arctanh((a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5661, 266, 63, 208}

$$-a \tanh^{-1}\left(\sqrt{a^2x^2+1}\right) - \frac{\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^2,x]
```

```
[Out] -(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x^2}\,dx &= -\frac{\sinh^{-1}(ax)}{x} + a\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sinh^{-1}(ax)}{x} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right)\\ &= -\frac{\sinh^{-1}(ax)}{x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a}\\ &= -\frac{\sinh^{-1}(ax)}{x} - a\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-a\tanh^{-1}\left(\sqrt{a^2x^2+1}\right) - \frac{\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^2,x]
```

```
[Out] -(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

fricas [B] time = 0.47, size = 90, normalized size = 3.33

$$-\frac{ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right) - (x-1)\log\left(ax+\sqrt{a^2x^2+1}\right) - x\log\left(-ax\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a*x*log(-a*x + sqrt(a^2*x^2 + 1)
- 1) - (x - 1)*log(a*x + sqrt(a^2*x^2 + 1)) - x*log(-a*x + sqrt(a^2*x^2 + 1
)))/x
```

giac [B] time = 0.13, size = 56, normalized size = 2.07

$$-\frac{1}{2}a\left(\log\left(\sqrt{a^2x^2+1}+1\right) - \log\left(\sqrt{a^2x^2+1}-1\right)\right) - \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*a*(log(sqrt(a^2*x^2 + 1) + 1) - log(sqrt(a^2*x^2 + 1) - 1)) - log(a*x
+ sqrt(a^2*x^2 + 1))/x
```

maple [A] time = 0.02, size = 30, normalized size = 1.11

$$a\left(-\frac{\operatorname{arcsinh}(ax)}{ax} - \operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^2,x)
```

```
[Out] a*(-arcsinh(a*x)/a/x-arctanh(1/(a^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.32, size = 22, normalized size = 0.81

$$-a\,\mathrm{arsinh}\left(\frac{1}{a|x|}\right)-\frac{\mathrm{arsinh}\,(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -a*arcsinh(1/(a*abs(x))) - arcsinh(a*x)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{asinh}\,(a\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/x^2,x)
```

```
[Out] int(asinh(a*x)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{asinh}\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**2,x)
```

```
[Out] Integral(asinh(a*x)/x**2, x)
```

3.8 $\int \frac{\sinh^{-1}(ax)}{x^3} dx$

Optimal. Leaf size=33

$$-\frac{a\sqrt{a^2x^2+1}}{2x}-\frac{\sinh^{-1}(ax)}{2x^2}$$

```
[Out] -1/2*arcsinh(a*x)/x^2-1/2*a*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5661, 264}

$$-\frac{a\sqrt{a^2x^2+1}}{2x}-\frac{\sinh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^3,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(2*x) - ArcSinh[a*x]/(2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x^3} dx &= -\frac{\sinh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int \frac{1}{x^2\sqrt{1+a^2x^2}} dx\\ &= -\frac{a\sqrt{1+a^2x^2}}{2x}-\frac{\sinh^{-1}(ax)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.85

$$-\frac{ax\sqrt{a^2x^2+1}+\sinh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^3,x]
```

```
[Out] -1/2*(a*x*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/x^2
```

fricas [A] time = 0.42, size = 36, normalized size = 1.09

$$-\frac{\sqrt{a^2x^2+1}\,ax+\log\left(ax+\sqrt{a^2x^2+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="fricas")
[Out] -1/2*(sqrt(a^2*x^2 + 1)*a*x + log(a*x + sqrt(a^2*x^2 + 1)))/x^2
```

giac [A] time = 0.14, size = 50, normalized size = 1.52

$$\frac{a|a|}{\left(x|a| - \sqrt{a^2x^2 + 1}\right)^2 - 1} - \frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="giac")
[Out] a*abs(a)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1) - 1/2*log(a*x + sqrt(a^2*x^
2 + 1))/x^2
```

maple [A] time = 0.02, size = 37, normalized size = 1.12

$$a^2\left(-\frac{\operatorname{arcsinh}(ax)}{2a^2x^2} - \frac{\sqrt{a^2x^2 + 1}}{2ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^3,x)
[Out] a^2*(-1/2*arcsinh(a*x)/a^2/x^2-1/2/a/x*(a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.31, size = 27, normalized size = 0.82

$$-\frac{\sqrt{a^2x^2 + 1}\,a}{2\,x} - \frac{\operatorname{arsinh}(ax)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="maxima")
[Out] -1/2*sqrt(a^2*x^2 + 1)*a/x - 1/2*arcsinh(a*x)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}(a\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/x^3,x)
[Out] int(asinh(a*x)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**3,x)
[Out] Integral(asinh(a*x)/x**3, x)
```

3.9 $\int \frac{\sinh^{-1}(ax)}{x^4} \, dx$

Optimal. Leaf size=54

$$-\frac{a\sqrt{a^2x^2+1}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{3x^3}$$

```
[Out] -1/3*arcsinh(a*x)/x^3+1/6*a^3*arctanh((a^2*x^2+1)^(1/2))-1/6*a*(a^2*x^2+1)^
(1/2)/x^2
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5661, 266, 51, 63, 208}

$$-\frac{a\sqrt{a^2x^2+1}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^4,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(6*x^2) - ArcSinh[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1
 + a^2*x^2]])/6
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^4}\,dx &= -\frac{\sinh^{-1}(ax)}{3x^3}+\frac{1}{3}a\int \frac{1}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\sinh^{-1}(ax)}{3x^3}+\frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\sinh^{-1}(ax)}{3x^3}-\frac{1}{12}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\sinh^{-1}(ax)}{3x^3}-\frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\sinh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$
-\frac{a\sqrt{a^2x^2+1}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^4,x]
```

```
[Out] -1/6*(a*Sqrt[1 + a^2*x^2])/x^2 - ArcSinh[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1
 + a^2*x^2]])/6
```

fricas [B] time = 0.44, size = 117, normalized size = 2.17

$$
\frac{a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+2\,x^3\log\left(-ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}\,ax}{6\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^3*x^3*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) + 2*x^3*log(-a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*a
*x + 2*(x^3 - 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x^3
```

giac [A] time = 0.12, size = 84, normalized size = 1.56

$$
\frac{a^4\log\left(\sqrt{a^2x^2+1}+1\right)-a^4\log\left(\sqrt{a^2x^2+1}-1\right)-\frac{2\sqrt{a^2x^2+1}\,a^2}{x^2}}{12\,a}-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{3\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a^4*log(sqrt(a^2*x^2 + 1) + 1) - a^4*log(sqrt(a^2*x^2 + 1) - 1) - 2*s
qrt(a^2*x^2 + 1)*a^2/x^2)/a - 1/3*log(a*x + sqrt(a^2*x^2 + 1))/x^3
```

maple [A] time = 0.02, size = 51, normalized size = 0.94

$$
a^3\left(-\frac{\operatorname{arcsinh}(ax)}{3a^3x^3}-\frac{\sqrt{a^2x^2+1}}{6a^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{6}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^4,x)
```

```
[Out] a^3*(-1/3*arcsinh(a*x)/a^3/x^3-1/6/a^2/x^2*(a^2*x^2+1)^(1/2)+1/6*arctanh(1/
(a^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.33, size = 43, normalized size = 0.80

$$\frac{1}{6}\left(a^2 \operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{\sqrt{a^2x^2+1}}{x^2}\right)a-\frac{\operatorname{arsinh}(ax)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x^2)*a - 1/3*arcsinh(a*x
)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/x^4,x)
```

```
[Out] int(asinh(a*x)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**4,x)
```

```
[Out] Integral(asinh(a*x)/x**4, x)
```

3.10 $\int \frac{\sinh^{-1}(ax)}{x^5} \, dx$

Optimal. Leaf size=56

$$-\frac{a\sqrt{a^2x^2+1}}{12x^3}+\frac{a^3\sqrt{a^2x^2+1}}{6x}-\frac{\sinh^{-1}(ax)}{4x^4}$$

```
[Out] -1/4*arcsinh(a*x)/x^4-1/12*a*(a^2*x^2+1)^(1/2)/x^3+1/6*a^3*(a^2*x^2+1)^(1/2
)/x
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5661, 271, 264}

$$\frac{a^3\sqrt{a^2x^2+1}}{6x}-\frac{a\sqrt{a^2x^2+1}}{12x^3}-\frac{\sinh^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^5,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(12*x^3) + (a^3*Sqrt[1 + a^2*x^2])/(6*x) - ArcSinh[a
*x]/(4*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^5} \, dx &= -\frac{\sinh^{-1}(ax)}{4x^4}+\frac{1}{4}a\int \frac{1}{x^4\sqrt{1+a^2x^2}} \, dx \\
&= -\frac{a\sqrt{1+a^2x^2}}{12x^3}-\frac{\sinh^{-1}(ax)}{4x^4}-\frac{1}{6}a^3\int \frac{1}{x^2\sqrt{1+a^2x^2}} \, dx \\
&= -\frac{a\sqrt{1+a^2x^2}}{12x^3}+\frac{a^3\sqrt{1+a^2x^2}}{6x}-\frac{\sinh^{-1}(ax)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.71

$$\frac{ax\sqrt{a^2x^2+1}\left(2a^2x^2-1\right)-3\sinh^{-1}(ax)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^5,x]
```

```
[Out] (a*x*Sqrt[1 + a^2*x^2]*(-1 + 2*a^2*x^2) - 3*ArcSinh[a*x])/(12*x^4)
```

fricas [A] time = 0.42, size = 49, normalized size = 0.88

$$\frac{\left(2\,a^3x^3 - ax\right)\sqrt{a^2x^2+1} - 3\,\log\left(ax + \sqrt{a^2x^2+1}\right)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*((2*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1) - 3*log(a*x + sqrt(a^2*x^2 + 1)))
/x^4
```

giac [A] time = 0.15, size = 77, normalized size = 1.38

$$\frac{\left(3\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)a^3|a|}{3\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^3} - \frac{\log\left(ax + \sqrt{a^2x^2+1}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)*a^3*abs(a)/((x*abs(a) - sqrt(a
^2*x^2 + 1))^2 - 1)^3 - 1/4*log(a*x + sqrt(a^2*x^2 + 1))/x^4
```

maple [A] time = 0.02, size = 56, normalized size = 1.00

$$a^4\left(-\frac{\operatorname{arcsinh}(ax)}{4a^4x^4} - \frac{\sqrt{a^2x^2+1}}{12a^3x^3} + \frac{\sqrt{a^2x^2+1}}{6ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^5,x)
```

```
[Out] a^4*(-1/4*arcsinh(a*x)/a^4/x^4-1/12/a^3/x^3*(a^2*x^2+1)^(1/2)+1/6/a/x*(a^2*
x^2+1)^(1/2))
```

maxima [A] time = 0.31, size = 49, normalized size = 0.88

$$\frac{1}{12}\left(\frac{2\sqrt{a^2x^2+1}\,a^2}{x} - \frac{\sqrt{a^2x^2+1}}{x^3}\right)a - \frac{\operatorname{arsinh}(ax)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(2*sqrt(a^2*x^2 + 1)*a^2/x - sqrt(a^2*x^2 + 1)/x^3)*a - 1/4*arcsinh(a*
x)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/x^5,x)
```

```
[Out] int(asinh(a*x)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**5,x)
```

```
[Out] Integral(asinh(a*x)/x**5, x)
```

3.11 $\int \frac{\sinh^{-1}(ax)}{x^6} \, dx$

Optimal. Leaf size=77

$$-\frac{a\sqrt{a^2x^2+1}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)+\frac{3a^3\sqrt{a^2x^2+1}}{40x^2}-\frac{\sinh^{-1}(ax)}{5x^5}$$

```
[Out] -1/5*arcsinh(a*x)/x^5-3/40*a^5*arctanh((a^2*x^2+1)^(1/2))-1/20*a*(a^2*x^2+1
)^(1/2)/x^4+3/40*a^3*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5661, 266, 51, 63, 208}

$$\frac{3a^3\sqrt{a^2x^2+1}}{40x^2}-\frac{a\sqrt{a^2x^2+1}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{a^2x^2+1}\right)-\frac{\sinh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/x^6,x]
```

```
[Out] -(a*Sqrt[1 + a^2*x^2])/(20*x^4) + (3*a^3*Sqrt[1 + a^2*x^2])/(40*x^2) - ArcS
inh[a*x]/(5*x^5) - (3*a^5*ArcTanh[Sqrt[1 + a^2*x^2]])/40
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x^6}\,dx &= -\frac{\sinh^{-1}(ax)}{5x^5}+\frac{1}{5}a\int \frac{1}{x^5\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sinh^{-1}(ax)}{5x^5}+\frac{1}{10}a\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+a^2x}}\,dx,x,x^2\right)\\ &= -\frac{a\sqrt{1+a^2x^2}}{20x^4}-\frac{\sinh^{-1}(ax)}{5x^5}-\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)\\ &= -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\sinh^{-1}(ax)}{5x^5}+\frac{1}{80}\left(3a^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\ &= -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\sinh^{-1}(ax)}{5x^5}+\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\ &= -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\sinh^{-1}(ax)}{5x^5}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1+a^2x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.64

$$-\frac{1}{5}a^5\sqrt{a^2x^2+1}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};a^2x^2+1\right)-\frac{\sinh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/x^6,x]
```

```
[Out] -1/5*ArcSinh[a*x]/x^5 - (a^5*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2, 3, 3/
2, 1 + a^2*x^2])/5
```

fricas [B] time = 0.45, size = 129, normalized size = 1.68

$$-\frac{3\,a^5x^5\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-3\,a^5x^5\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-8\,x^5\log\left(-ax+\sqrt{a^2x^2+1}\right)-8\left(x^5-1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-\left(3\,a^3x^3-2\,ax\right)\sqrt{a^2x^2+1}}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/40*(3*a^5*x^5*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*a^5*x^5*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) - 8*x^5*log(-a*x + sqrt(a^2*x^2 + 1)) - 8*(x^5 - 1)*l
og(a*x + sqrt(a^2*x^2 + 1)) - (3*a^3*x^3 - 2*a*x)*sqrt(a^2*x^2 + 1))/x^5
```

giac [A] time = 0.12, size = 107, normalized size = 1.39

$$-\frac{3\,a^6\log\left(\sqrt{a^2x^2+1}+1\right)-3\,a^6\log\left(\sqrt{a^2x^2+1}-1\right)-\frac{2\left(3\left(a^2x^2+1\right)^{\frac{3}{2}}a^6-5\sqrt{a^2x^2+1}\,a^6\right)}{a^4x^4}}{80\,a}-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] -1/80*(3*a^6*log(sqrt(a^2*x^2 + 1) + 1) - 3*a^6*log(sqrt(a^2*x^2 + 1) - 1)
- 2*(3*(a^2*x^2 + 1)^(3/2)*a^6 - 5*sqrt(a^2*x^2 + 1)*a^6)/(a^4*x^4))/a - 1/
5*log(a*x + sqrt(a^2*x^2 + 1))/x^5
```

maple [A] time = 0.03, size = 70, normalized size = 0.91

$$a^{5}\left(-\frac{\operatorname{arcsinh}(ax)}{5a^{5}x^{5}}-\frac{\sqrt{a^{2}x^{2}+1}}{20a^{4}x^{4}}+\frac{3\sqrt{a^{2}x^{2}+1}}{40a^{2}x^{2}}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^6,x)
```

```
[Out] a^5*(-1/5*arcsinh(a*x)/a^5/x^5-1/20/a^4/x^4*(a^2*x^2+1)^(1/2)+3/40/a^2/x^2*
(a^2*x^2+1)^(1/2)-3/40*arctanh(1/(a^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.30, size = 63, normalized size = 0.82

$$-\frac{1}{40}\left(3\,a^{4}\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{3\,\sqrt{a^{2}x^{2}+1}\,a^{2}}{x^{2}}+\frac{2\,\sqrt{a^{2}x^{2}+1}}{x^{4}}\right)a-\frac{\operatorname{arsinh}(ax)}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/40*(3*a^4*arcsinh(1/(a*abs(x))) - 3*sqrt(a^2*x^2 + 1)*a^2/x^2 + 2*sqrt(a
^2*x^2 + 1)/x^4)*a - 1/5*arcsinh(a*x)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)/x^6,x)
```

```
[Out] int(asinh(a*x)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**6,x)
```

```
[Out] Integral(asinh(a*x)/x**6, x)
```

3.12 $\int x^4 \sinh^{-1}(ax)^2\,dx$

Optimal. Leaf size=117

$$\frac{16x}{75a^4}-\frac{8x^3}{225a^2}-\frac{2x^4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{25a}-\frac{16\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{75a^5}+\frac{8x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{75a^3}+\frac{1}{5}x^5\sinh^{-1}(ax)^2$$

```
[Out] 16/75*x/a^4-8/225*x^3/a^2+2/125*x^5+1/5*x^5*arcsinh(a*x)^2-16/75*arcsinh(a*
x)*(a^2*x^2+1)^(1/2)/a^5+8/75*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-2/25*x
^4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5661, 5758, 5717, 8, 30}

$$-\frac{8x^3}{225a^2}-\frac{2x^4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{25a}+\frac{8x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{75a^3}-\frac{16\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{75a^5}+\frac{16x}{75a^4}+\frac{1}{5}x^5\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^2,x]
```

```
[Out] (16*x)/(75*a^4) - (8*x^3)/(225*a^2) + (2*x^5)/125 - (16*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x])/(75*a^5) + (8*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(75*a^3) - (
2*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(25*a) + (x^5*ArcSinh[a*x]^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \sinh^{-1}(ax)^2\,dx &= \frac{1}{5}x^5\sinh^{-1}(ax)^2 - \frac{1}{5}(2a)\int \frac{x^5\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{2x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^2 + \frac{2\int x^4\,dx}{25} + \frac{8\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{25a}\\
&= \frac{2x^5}{125} + \frac{8x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^2 - \frac{16}{\cdots}\\
&= -\frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{75a^5} + \frac{8x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1+a^2x\cdots}}{25\cdots}\\
&= \frac{16x}{75a^4} - \frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{75a^5} + \frac{8x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{75a^3} - \frac{2x^4\sqrt{1\cdots}}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.64

$$
\frac{\frac{240x}{a^4} - \frac{40x^3}{a^2} - \frac{30\sqrt{a^2x^2+1}\left(3a^4x^4-4a^2x^2+8\right)\sinh^{-1}(ax)}{a^5} + 225x^5\sinh^{-1}(ax)^2 + 18x^5}{1125}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x]^2,x]
```

```
[Out] ((240*x)/a^4 - (40*x^3)/a^2 + 18*x^5 - (30*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2
 + 3*a^4*x^4)*ArcSinh[a*x])/a^5 + 225*x^5*ArcSinh[a*x]^2)/1125
```

fricas [A] time = 0.42, size = 99, normalized size = 0.85

$$
\frac{225\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 18\,a^5x^5 - 40\,a^3x^3 - 30\left(3\,a^4x^4 - 4\,a^2x^2 + 8\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1\cdots}\right.}{1125\,a^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^2 + 18*a^5*x^5 - 40*a^3*x^
3 - 30*(3*a^4*x^4 - 4*a^2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 + 1)) + 240*a*x)/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.38, size = 103, normalized size = 0.88

$$
\frac{\frac{a^5x^5\operatorname{arcsinh}(ax)^2}{5} - \frac{16\sqrt{a^2x^2+1}\ \operatorname{arcsinh}(ax)}{75} - \frac{2a^4x^4\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{25} + \frac{8\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\ a^2x^2}{75} + \frac{16ax}{75} + \frac{2a^5x^5}{125} - \frac{8a^3x^3}{225}}{a^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^2,x)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)^2-16/75*(a^2*x^2+1)^(1/2)*arcsinh(a*x)-2/25
*a^4*x^4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+8/75*arcsinh(a*x)*(a^2*x^2+1)^(1/2)
*a^2*x^2+16/75*a*x+2/125*a^5*x^5-8/225*a^3*x^3)
```

maxima [A] time = 0.34, size = 99, normalized size = 0.85

$$\frac{1}{5}x^5\operatorname{arsinh}(ax)^2-\frac{2}{75}\left(\frac{3\sqrt{a^2x^2+1}\,x^4}{a^2}-\frac{4\sqrt{a^2x^2+1}\,x^2}{a^4}+\frac{8\sqrt{a^2x^2+1}}{a^6}\right)a\operatorname{arsinh}(ax)+\frac{2\left(9\,a^4x^5-20\,a^2x^3+120\right.}{1125\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^2 - 2/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x) + 2/1125*(9*a^4*x^5
 - 20*a^2*x^3 + 120*x)/a^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\operatorname{asinh}(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asinh(a*x)^2,x)
```

```
[Out] int(x^4*asinh(a*x)^2, x)
```

sympy [A] time = 3.03, size = 114, normalized size = 0.97

$$\begin{cases}\frac{x^5\operatorname{asinh}^2(ax)}{5}+\frac{2x^5}{125}-\frac{2x^4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{25a}-\frac{8x^3}{225a^2}+\frac{8x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{75a^3}+\frac{16x}{75a^4}-\frac{16\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{75a^5} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**2/5 + 2*x**5/125 - 2*x**4*sqrt(a**2*x**2 + 1)*a
sinh(a*x)/(25*a) - 8*x**3/(225*a**2) + 8*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x
)/(75*a**3) + 16*x/(75*a**4) - 16*sqrt(a**2*x**2 + 1)*asinh(a*x)/(75*a**5),
 Ne(a, 0)), (0, True))
```

3.13 $\int x^3 \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=96

$$-\frac{3\sinh^{-1}(ax)^2}{32a^4}-\frac{3x^2}{32a^2}-\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{8a}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{16a^3}+\frac{1}{4}x^4\sinh^{-1}(ax)^2+\frac{x^4}{32}$$

```
[Out] -3/32*x^2/a^2+1/32*x^4-3/32*arcsinh(a*x)^2/a^4+1/4*x^4*arcsinh(a*x)^2+3/16*
x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-1/8*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)
/a
```

Rubi [A] time = 0.17, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5661, 5758, 5675, 30}

$$-\frac{3x^2}{32a^2}-\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{8a}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{16a^3}-\frac{3\sinh^{-1}(ax)^2}{32a^4}+\frac{1}{4}x^4\sinh^{-1}(ax)^2+\frac{x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^2,x]
```

```
[Out] (-3*x^2)/(32*a^2) + x^4/32 + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(16*a^3)
- (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a) - (3*ArcSinh[a*x]^2)/(32*a^4)
+ (x^4*ArcSinh[a*x]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^2\,dx = \frac{1}{4}x^4 \sinh^{-1}(ax)^2 - \frac{1}{2}a\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a} + \frac{1}{4}x^4 \sinh^{-1}(ax)^2 + \frac{\int x^3\,dx}{8} + \frac{3\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a}$$

$$= \frac{x^4}{32} + \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a} + \frac{1}{4}x^4 \sinh^{-1}(ax)^2 - \frac{3\int \frac{\sinh^{-1}}{\sqrt{1+a^2}}}{16a}$$

$$= -\frac{3x^2}{32a^2} + \frac{x^4}{32} + \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{16a^3} - \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{8a} - \frac{3\sinh^{-1}(ax)^2}{32a^4} + \frac{1}{4}x$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.75

$$\frac{\left(8a^4x^4-3\right)\sinh^{-1}(ax)^2 + a^2x^2\left(a^2x^2-3\right) - 2ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x]^2,x]
```

```
[Out] (a^2*x^2*(-3 + a^2*x^2) - 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[
a*x] + (-3 + 8*a^4*x^4)*ArcSinh[a*x]^2)/(32*a^4)
```

fricas [A] time = 0.41, size = 92, normalized size = 0.96

$$\frac{a^4x^4 - 3\,a^2x^2 + \left(8\,a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 2\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{32\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(a^4*x^4 - 3*a^2*x^2 + (8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^2
- 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.38, size = 87, normalized size = 0.91

$$\frac{\frac{a^4x^4\operatorname{arcsinh}(ax)^2}{4} - \frac{a^3x^3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{8} + \frac{3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{16} - \frac{3\operatorname{arcsinh}(ax)^2}{32} + \frac{a^4x^4}{32} - \frac{3a^2x^2}{32} - \frac{3}{32}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^2,x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)^2-1/8*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2
)+3/16*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-3/32*arcsinh(a*x)^2+1/32*a^4*x^4-
3/32*a^2*x^2-3/32)
```

maxima [A] time = 0.33, size = 109, normalized size = 1.14

$$\frac{1}{4}x^4 \operatorname{arsinh}(ax)^2+\frac{1}{32}\left(\frac{x^4}{a^2}-\frac{3x^2}{a^4}+\frac{3\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^6}\right)a^2-\frac{1}{16}\left(\frac{2\sqrt{a^2x^2+1}\,x^3}{a^2}-\frac{3\sqrt{a^2x^2+1}\,x}{a^4}+\frac{3\operatorname{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x)^2 + 1/32*(x^4/a^2 - 3*x^2/a^4 + 3*log(a*x + sqrt(a^2*x
^2 + 1))^2/a^6)*a^2 - 1/16*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 +
1)*x/a^4 + 3*arcsinh(a*x)/a^5)*a*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asinh}(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x)^2,x)
```

```
[Out] int(x^3*asinh(a*x)^2, x)
```

sympy [A] time = 1.86, size = 90, normalized size = 0.94

$$\begin{cases} \frac{x^4\operatorname{asinh}^2(ax)}{4}+\frac{x^4}{32}-\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{8a}-\frac{3x^2}{32a^2}+\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{16a^3}-\frac{3\operatorname{asinh}^2(ax)}{32a^4} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**2/4 + x**4/32 - x**3*sqrt(a**2*x**2 + 1)*asinh(
a*x)/(8*a) - 3*x**2/(32*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(16*a**3
) - 3*asinh(a*x)**2/(32*a**4), Ne(a, 0)), (0, True))
```

3.14 $\int x^2 \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=80

$$-\frac{2x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a}-\frac{4x}{9a^2}+\frac{4\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a^3}+\frac{1}{3}x^3\sinh^{-1}(ax)^2+\frac{2x^3}{27}$$

```
[Out] -4/9*x/a^2+2/27*x^3+1/3*x^3*arcsinh(a*x)^2+4/9*arcsinh(a*x)*(a^2*x^2+1)^(1/
2)/a^3-2/9*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{5661, 5758, 5717, 8, 30\}$

$$-\frac{2x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a}+\frac{4\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a^3}-\frac{4x}{9a^2}+\frac{1}{3}x^3\sinh^{-1}(ax)^2+\frac{2x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^2,x]
```

```
[Out] (-4*x)/(9*a^2) + (2*x^3)/27 + (4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^3) -
(2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a) + (x^3*ArcSinh[a*x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\sinh^{-1}(ax)^2\,dx = \frac{1}{3}x^3\sinh^{-1}(ax)^2 - \frac{1}{3}(2a)\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^2 + \frac{2\int x^2\,dx}{9} + \frac{4\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{9a}$$
$$= \frac{2x^3}{27} + \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^2 - \frac{4\int 1}{9a^2}$$
$$= -\frac{4x}{9a^2} + \frac{2x^3}{27} + \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a^3} - \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^2$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.74

$$\frac{1}{27}\left(2x\left(x^2-\frac{6}{a^2}\right)-\frac{6\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a^3}+9x^3\sinh^{-1}(ax)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x]^2,x]
```

```
[Out] (2*x*(-6/a^2 + x^2) - (6*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^3
 + 9*x^3*ArcSinh[a*x]^2)/27
```

fricas [A] time = 0.42, size = 82, normalized size = 1.02

$$\frac{9\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a^3x^3-6\,\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-12\,ax}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a^3*x^3 - 6*sqrt(a^2*x^2
 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1)) - 12*a*x)/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.38, size = 72, normalized size = 0.90

$$\frac{\frac{a^3x^3\operatorname{arcsinh}(ax)^2}{3}+\frac{4\sqrt{a^2x^2+1}\,\operatorname{arcsinh}(ax)}{9}-\frac{2\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,a^2x^2}{9}-\frac{4ax}{9}+\frac{2a^3x^3}{27}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)^2+4/9*(a^2*x^2+1)^(1/2)*arcsinh(a*x)-2/9*ar
csinh(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-4/9*a*x+2/27*a^3*x^3)
```

maxima [A] time = 0.33, size = 70, normalized size = 0.88

$$\frac{1}{3}x^3 \operatorname{arsinh}(ax)^2 - \frac{2}{9}a\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax) + \frac{2\left(a^2x^3-6x\right)}{27a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^2 - 2/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x) + 2/27*(a^2*x^3 - 6*x)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asinh}(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x)^2,x)
```

```
[Out] int(x^2*asinh(a*x)^2, x)
```

sympy [A] time = 0.86, size = 76, normalized size = 0.95

$$\begin{cases} \frac{x^3 \operatorname{asinh}^2(ax)}{3} + \frac{2x^3}{27} - \frac{2x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a} - \frac{4x}{9a^2} + \frac{4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**2/3 + 2*x**3/27 - 2*x**2*sqrt(a**2*x**2 + 1)*as
inh(a*x)/(9*a) - 4*x/(9*a**2) + 4*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**3),
Ne(a, 0)), (0, True))
```

3.15 $\int x \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=59

$$-\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a}+\frac{\sinh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^2+\frac{x^2}{4}$$

```
[Out] 1/4*x^2+1/4*arcsinh(a*x)^2/a^2+1/2*x^2*arcsinh(a*x)^2-1/2*x*arcsinh(a*x)*(a
^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5661, 5758, 5675, 30}

$$-\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a}+\frac{\sinh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^2+\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^2,x]
```

```
[Out] x^2/4 - (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a) + ArcSinh[a*x]^2/(4*a^2) +
 (x^2*ArcSinh[a*x]^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x\sinh^{-1}(ax)^2\,dx = \frac{1}{2}x^2\sinh^{-1}(ax)^2 - a\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a} + \frac{1}{2}x^2\sinh^{-1}(ax)^2 + \frac{\int x\,dx}{2} + \frac{\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a}$$
$$= \frac{x^2}{4} - \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a} + \frac{\sinh^{-1}(ax)^2}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^2$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.90

$$\frac{a^2x^2 - 2ax\sqrt{a^2x^2+1}\sinh^{-1}(ax) + \left(2a^2x^2+1\right)\sinh^{-1}(ax)^2}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x]^2,x]
```

```
[Out] (a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (1 + 2*a^2*x^2)*ArcSinh[a
*x]^2)/(4*a^2)
```

fricas [A] time = 0.41, size = 73, normalized size = 1.24

$$\frac{a^2x^2 - 2\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right) + \left(2\,a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + (2*a^
2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.08, size = 59, normalized size = 1.00

$$\frac{\frac{\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^2}{2} - \frac{\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{2} - \frac{\operatorname{arcsinh}(ax)^2}{4} + \frac{a^2x^2}{4} + \frac{1}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^2,x)
```

```
[Out] 1/a^2*(1/2*(a^2*x^2+1)*arcsinh(a*x)^2-1/2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*
x-1/4*arcsinh(a*x)^2+1/4*a^2*x^2+1/4)
```

maxima [A] time = 0.34, size = 81, normalized size = 1.37

$$\frac{1}{2}\,x^2\,\mathrm{arsinh}\,(ax)^2+\frac{1}{4}\,a^2\left(\frac{x^2}{a^2}-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^4}\right)-\frac{1}{2}\,a\left(\frac{\sqrt{a^2x^2+1}\,x}{a^2}-\frac{\mathrm{arsinh}\,(ax)}{a^3}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a*x)^2 + 1/4*a^2*(x^2/a^2 - log(a*x + sqrt(a^2*x^2 + 1))^2/
a^4) - 1/2*a*(sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)*arcsinh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{asinh}\,(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x)^2,x)
```

```
[Out] int(x*asinh(a*x)^2, x)
```

sympy [A] time = 0.43, size = 51, normalized size = 0.86

$$\begin{cases} \frac{x^2\,\mathrm{asinh}^2\,(ax)}{2}+\frac{x^2}{4}-\frac{x\sqrt{a^2x^2+1}\,\mathrm{asinh}\,(ax)}{2a}+\frac{\mathrm{asinh}^2\,(ax)}{4a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**2/2 + x**2/4 - x*sqrt(a**2*x**2 + 1)*asinh(a*x)
/(2*a) + asinh(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

3.16 $\int \sinh^{-1}(ax)^2\,dx$

Optimal. Leaf size=34

$$-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2+2x$$

```
[Out] 2*x+x*arcsinh(a*x)^2-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5653, 5717, 8}

$$-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2+2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + x*ArcSinh[a*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}(ax)^2\,dx &= x\sinh^{-1}(ax)^2-(2a)\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2+2\int 1\,dx\\
&= 2x-\frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$-\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^2+2x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + x*ArcSinh[a*x]^2
```

fricas [A] time = 0.41, size = 59, normalized size = 1.74

$$\frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,ax-2\,\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*x - 2*sqrt(a^2*x^2 + 1)*log(a*x +
 sqrt(a^2*x^2 + 1)))/a
```

giac [A] time = 0.13, size = 62, normalized size = 1.82

$$x\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x + s
qrt(a^2*x^2 + 1))/a^2)
```

maple [A] time = 0.09, size = 36, normalized size = 1.06

$$\frac{ax\,\mathrm{arcsinh}\,(ax)^2-2\sqrt{a^2x^2+1}\,\,\mathrm{arcsinh}\,(ax)+2ax}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2,x)
```

```
[Out] 1/a*(a*x*arcsinh(a*x)^2-2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)+2*a*x)
```

maxima [A] time = 0.32, size = 32, normalized size = 0.94

$$x\,\mathrm{arsinh}\,(ax)^2+2\,x-\frac{2\,\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^2 + 2*x - 2*sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{asinh}\,(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2,x)
```

```
[Out] int(asinh(a*x)^2, x)
```

sympy [A] time = 0.19, size = 32, normalized size = 0.94

$$\begin{cases} x\operatorname{asinh}^2(ax) + 2x - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2,x)
```

```
[Out] Piecewise((x*asinh(a*x)**2 + 2*x - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/a, Ne(a
, 0)), (0, True))
```

3.17 $\int \frac{\sinh^{-1}(ax)^2}{x} dx$

Optimal. Leaf size=60

$$\sinh^{-1}(ax)\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\text{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)-\frac{1}{3}\sinh^{-1}(ax)^3+\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

```
[Out] -1/3*arcsinh(a*x)^3+arcsinh(a*x)^2*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+arcsinh(
a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-1/2*polylog(3,(a*x+(a^2*x^2+1)^(1
/2))^2)
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5659, 3716, 2190, 2531, 2282, 6589}

$$\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{3}\sinh^{-1}(ax)^3+\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x,x]
```

```
[Out] -ArcSinh[a*x]^3/3 + ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + ArcSinh[a*
x]*PolyLog[2, E^(2*ArcSinh[a*x])] - PolyLog[3, E^(2*ArcSinh[a*x])]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
```

```
0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x}\,dx &= \text{Subst}\left(\int x^2\coth(x)\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 - 2\,\text{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) - 2\,\text{Subst}\left(\int x\log\left(1-e^{2x}\right)\,dx, x, \sinh^{-1}\right.\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax)\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \text{Subst}\left(\right.\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax)\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{1}{2}\text{Subst}\\
&= -\frac{1}{3}\sinh^{-1}(ax)^3 + \sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \sinh^{-1}(ax)\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{1}{2}\text{Li}_3\left(e^{2}\right.
\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.00

$$\sinh^{-1}(ax)\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-\frac{1}{2}\text{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)-\frac{1}{3}\sinh^{-1}(ax)^3+\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/x,x]
```

```
[Out] -1/3*ArcSinh[a*x]^3 + ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + ArcSinh[
a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - PolyLog[3, E^(2*ArcSinh[a*x])]/2
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x, x)
```

maple [A] time = 0.17, size = 151, normalized size = 2.52

$$-\frac{\operatorname{arcsinh}(ax)^3}{3}+\operatorname{arcsinh}(ax)^2\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+2\operatorname{arcsinh}(ax)\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2+1}\right)-2\operatorname{polyl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x,x)
```

```
[Out] -1/3*arcsinh(a*x)^3+arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*
x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))+arcs
inh(a*x)^2*ln(a*x+(a^2*x^2+1)^(1/2)+1)+2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x
^2+1)^(1/2))-2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{arsinh}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\operatorname{asinh}(a\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/x,x)
```

```
[Out] int(asinh(a*x)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x,x)
```

```
[Out] Integral(asinh(a*x)**2/x, x)
```

3.18 $\int \frac{\sinh^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=50

$$-2a\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+2a\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sinh^{-1}(ax)^2}{x}-4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -arcsinh(a*x)^2/x-4*a*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-2*a*polyl
og(2,-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.10, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5661, 5760, 4182, 2279, 2391}

$$-2a\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+2a\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sinh^{-1}(ax)^2}{x}-4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x^2,x]
```

```
[Out] -(ArcSinh[a*x]^2/x) - 4*a*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - 2*a*PolyLo
g[2, -E^ArcSinh[a*x]] + 2*a*PolyLog[2, E^ArcSinh[a*x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{x^2}\,dx = -\frac{\sinh^{-1}(ax)^2}{x} + (2a)\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{\sinh^{-1}(ax)^2}{x} + (2a)\,\text{Subst}\left(\int x\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -\frac{\sinh^{-1}(ax)^2}{x} - 4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - (2a)\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \text{sinl}\right.$$
$$= -\frac{\sinh^{-1}(ax)^2}{x} - 4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - (2a)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh}\right.$$
$$= -\frac{\sinh^{-1}(ax)^2}{x} - 4a\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2a\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2a\text{Li}_2\left(e^{\sinh^{-1}(a}\right.$$

Mathematica [A] time = 0.21, size = 75, normalized size = 1.50

$$a\left(2\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) - 2\text{Li}_2\left(e^{-\sinh^{-1}(ax)}\right) - \sinh^{-1}(ax)\left(\frac{\sinh^{-1}(ax)}{ax} - 2\log\left(1-e^{-\sinh^{-1}(ax)}\right) + 2\log\left(e^{-\sinh^{-1}}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/x^2,x]
```

```
[Out] a*(-(ArcSinh[a*x]*(ArcSinh[a*x]/(a*x) - 2*Log[1 - E^(-ArcSinh[a*x])] + 2*Lo
g[1 + E^(-ArcSinh[a*x])])) + 2*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*PolyLog[2
, E^(-ArcSinh[a*x])])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\left(ax\right)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\left(ax\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x^2, x)
```

maple [A] time = 0.24, size = 107, normalized size = 2.14

$$-\frac{\text{arcsinh}\left(ax\right)^2}{x} + 2a\,\text{arcsinh}\left(ax\right)\ln\left(1-ax-\sqrt{a^2x^2+1}\right) + 2a\,\text{polylog}\left(2, ax+\sqrt{a^2x^2+1}\right) - 2a\,\text{arcsinh}\left(ax\right)\text{ln}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^2,x)
```

```
[Out] -arcsinh(a*x)^2/x+2*a*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(
2,a*x+(a^2*x^2+1)^(1/2))-2*a*arcsinh(a*x)*ln(a*x+(a^2*x^2+1)^(1/2)+1)-2*a*p
olylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{x}+\int\frac{2\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^3x^4+ax^2+\left(a^2x^3+x\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^4 + a*x^2 + (a^2*x^3 + x)*
sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\operatorname{asinh}(a\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/x^2,x)
```

```
[Out] int(asinh(a*x)^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**2,x)
```

```
[Out] Integral(asinh(a*x)**2/x**2, x)
```

3.19 $\int \frac{\sinh^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=43

$$-\frac{a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}+a^2\log(x)-\frac{\sinh^{-1}(ax)^2}{2x^2}$$

```
[Out] -1/2*arcsinh(a*x)^2/x^2+a^2*ln(x)-a*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5661, 5723, 29}

$$-\frac{a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}+a^2\log(x)-\frac{\sinh^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x^3,x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) - ArcSinh[a*x]^2/(2*x^2) + a^2*Log[
x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x^3}\,dx &= -\frac{\sinh^{-1}(ax)^2}{2x^2}+a\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}-\frac{\sinh^{-1}(ax)^2}{2x^2}+a^2\int \frac{1}{x}\,dx\\
&= -\frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{x}-\frac{\sinh^{-1}(ax)^2}{2x^2}+a^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 1.00

$$-\frac{a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{x}+a^2\log(x)-\frac{\sinh^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/x^3,x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) - ArcSinh[a*x]^2/(2*x^2) + a^2*Log[
x]
```

fricas [A] time = 0.42, size = 67, normalized size = 1.56

$$\frac{2\,a^2x^2\log(x) - 2\,\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right) - \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2*log(x) - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)
) - log(a*x + sqrt(a^2*x^2 + 1))^2)/x^2
```

giac [B] time = 0.17, size = 98, normalized size = 2.28

$$-\left(a\log\left(-x|a|+\sqrt{a^2x^2+1}\right) - a\log\left(|x|\right) - \frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}\right)a - \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] -(a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) - a*log(abs(x)) - 2*abs(a)*log(a*x +
 sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1))*a - 1/2*log(a*x
 + sqrt(a^2*x^2 + 1))^2/x^2
```

maple [A] time = 0.36, size = 67, normalized size = 1.56

$$-a^2\operatorname{arcsinh}(ax) - \frac{a\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{x} - \frac{\operatorname{arcsinh}(ax)^2}{2x^2} + a^2\ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^3,x)
```

```
[Out] -a^2*arcsinh(a*x)-a*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x-1/2*arcsinh(a*x)^2/x^2
+a^2*ln((a*x+(a^2*x^2+1)^(1/2))^2-1)
```

maxima [A] time = 0.31, size = 39, normalized size = 0.91

$$a^2\log(x) - \frac{\sqrt{a^2x^2+1}\,a\operatorname{arsinh}(ax)}{x} - \frac{\operatorname{arsinh}(ax)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*log(x) - sqrt(a^2*x^2 + 1)*a*arcsinh(a*x)/x - 1/2*arcsinh(a*x)^2/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/x^3,x)
```

```
[Out] int(asinh(a*x)^2/x^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**3,x)
```

```
[Out] Integral(asinh(a*x)**2/x**3, x)
```

3.20 $\int \frac{\sinh^{-1}(ax)^2}{x^4} \, dx$

Optimal. Leaf size=99

$$\frac{1}{3}a^3\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-\frac{1}{3}a^3\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)+\frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\frac{a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3x^2}-\frac{a^2}{3x}-\text{sinh}$$

```
[Out] -1/3*a^2/x-1/3*arcsinh(a*x)^2/x^3+2/3*a^3*arcsinh(a*x)*arctanh(a*x+(a^2*x^2
+1)^(1/2))+1/3*a^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-1/3*a^3*polylog(2,a*x+
(a^2*x^2+1)^(1/2))-1/3*a*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.16, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5661, 5747, 5760, 4182, 2279, 2391, 30}

$$\frac{1}{3}a^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{3}a^3\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{3x^2}-\frac{a^2}{3x}+\frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x^4,x]
```

```
[Out] -a^2/(3*x) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*x^2) - ArcSinh[a*x]^2/(3
*x^3) + (2*a^3*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]])/3 + (a^3*PolyLog[2, -E
^ArcSinh[a*x]])/3 - (a^3*PolyLog[2, E^ArcSinh[a*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x^4}\,dx &= -\frac{\sinh^{-1}(ax)^2}{3x^3}+\frac{1}{3}(2a)\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2}-\frac{\sinh^{-1}(ax)^2}{3x^3}+\frac{1}{3}a^2\int \frac{1}{x^2}\,dx-\frac{1}{3}a^3\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2}-\frac{\sinh^{-1}(ax)^2}{3x^3}-\frac{1}{3}a^3\,\mathrm{Subst}\left(\int x\,\mathrm{csch}(x)\,dx,x,\sinh^{-1}(ax\right.\\
&= -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2}-\frac{\sinh^{-1}(ax)^2}{3x^3}+\frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{3}a\\
&= -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2}-\frac{\sinh^{-1}(ax)^2}{3x^3}+\frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{3}a\\
&= -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x^2}-\frac{\sinh^{-1}(ax)^2}{3x^3}+\frac{2}{3}a^3\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{3}a
\end{aligned}$$

Mathematica [A] time = 0.53, size = 125, normalized size = 1.26

$$-\frac{a^3x^3\mathrm{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right)-a^3x^3\mathrm{Li}_2\left(e^{-\sinh^{-1}(ax)}\right)+a^3x^3\sinh^{-1}(ax)\log\left(1-e^{-\sinh^{-1}(ax)}\right)-a^3x^3\sinh^{-1}(ax)\log}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/x^4,x]
```

```
[Out] -1/3*(a^2*x^2 + a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2 + a^3*x
^3*ArcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] - a^3*x^3*ArcSinh[a*x]*Log[1 + E
^(-ArcSinh[a*x])] + a^3*x^3*PolyLog[2, -E^(-ArcSinh[a*x])] - a^3*x^3*PolyLo
g[2, E^(-ArcSinh[a*x])])/x^3
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{arsinh}\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x^4, x)
```

maple [A] time = 0.48, size = 144, normalized size = 1.45

$$-\frac{a\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{3x^2}-\frac{a^2}{3x}-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}-\frac{a^3\operatorname{arcsinh}(ax)\ln\left(1-ax-\sqrt{a^2x^2+1}\right)}{3}-\frac{a^3\operatorname{polylog}\left(2,ax+\sqrt{\right.}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^4,x)
```

```
[Out] -1/3*a*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x^2-1/3*a^2/x-1/3*arcsinh(a*x)^2/x^3-
1/3*a^3*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/3*a^3*polylog(2,a*x+(a^2
*x^2+1)^(1/2))+1/3*a^3*arcsinh(a*x)*ln(a*x+(a^2*x^2+1)^(1/2)+1)+1/3*a^3*pol
ylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{3\,x^3}+\int\frac{2\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{3\left(a^3x^6+ax^4+\left(a^2x^5+x^3\right)\sqrt{a^2x^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^2/x^3 + integrate(2/3*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^6 + a*x^4 + (a^2*x
^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/x^4,x)
```

```
[Out] int(asinh(a*x)^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**4,x)
```

```
[Out] Integral(asinh(a*x)**2/x**4, x)
```

3.21 $\int \frac{\sinh^{-1}(ax)^2}{x^5} \, dx$

Optimal. Leaf size=85

$$-\frac{1}{3}a^4 \log(x) - \frac{a^2}{12x^2} - \frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3x} - \frac{\sinh^{-1}(ax)^2}{4x^4}$$

```
[Out] -1/12*a^2/x^2-1/4*arcsinh(a*x)^2/x^4-1/3*a^4*ln(x)-1/6*a*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)/x^3+1/3*a^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{5661, 5747, 5723, 29, 30\}$

$$-\frac{a^2}{12x^2} + \frac{a^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3x} - \frac{a\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{6x^3} - \frac{1}{3}a^4 \log(x) - \frac{\sinh^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/x^5,x]
```

```
[Out] -a^2/(12*x^2) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(6*x^3) + (a^3*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x])/(3*x) - ArcSinh[a*x]^2/(4*x^4) - (a^4*Log[x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
```

```
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{x^5}\,dx = -\frac{\sinh^{-1}(ax)^2}{4x^4} + \frac{1}{2}a\int \frac{\sinh^{-1}(ax)}{x^4\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{6x^3} - \frac{\sinh^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^2\int \frac{1}{x^3}\,dx - \frac{1}{3}a^3\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{a^2}{12x^2} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x} - \frac{\sinh^{-1}(ax)^2}{4x^4} - \frac{1}{3}a^4\int \frac{1}{x}\,dx$$
$$= -\frac{a^2}{12x^2} - \frac{a\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{6x^3} + \frac{a^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3x} - \frac{\sinh^{-1}(ax)^2}{4x^4} - \frac{1}{3}a^4\log(x)$$

Mathematica [A] time = 0.07, size = 64, normalized size = 0.75

$$-\frac{4a^4x^4\log(x) + a^2x^2 - 2ax\sqrt{a^2x^2+1}\left(2a^2x^2-1\right)\sinh^{-1}(ax) + 3\sinh^{-1}(ax)^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/x^5,x]
```

```
[Out] -1/12*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*(-1 + 2*a^2*x^2)*ArcSinh[a*x] + 3*
ArcSinh[a*x]^2 + 4*a^4*x^4*Log[x])/x^4
```

fricas [A] time = 0.43, size = 85, normalized size = 1.00

$$-\frac{4\,a^4x^4\log(x) + a^2x^2 - 2\left(2\,a^3x^3 - ax\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right) + 3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a^4*x^4*log(x) + a^2*x^2 - 2*(2*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1)*l
og(a*x + sqrt(a^2*x^2 + 1)) + 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/x^4
```

giac [B] time = 0.25, size = 148, normalized size = 1.74

$$-\frac{1}{12}\left(2\,a^3\log\left(x^2\right) - 4\,a^3\log\left(-x|a| + \sqrt{a^2x^2+1}\right) - \frac{8\left(3\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)a^2|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^3} - \frac{2}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(2*a^3*log(x^2) - 4*a^3*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) - 8*(3*(x*
abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)*a^2*abs(a)*log(a*x + sqrt(a^2*x^2 + 1))/
((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^3 - (2*a^3*x^2 - a)/x^2)*a - 1/4*log
(a*x + sqrt(a^2*x^2 + 1))^2/x^4
```

maple [A] time = 0.45, size = 99, normalized size = 1.16

$$\frac{a^4\operatorname{arcsinh}(ax)}{3} + \frac{a^3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{3x} - \frac{a\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{6x^3} - \frac{a^2}{12x^2} - \frac{\operatorname{arcsinh}(ax)^2}{4x^4} - \frac{a^4\ln\left(\left(ax+\sqrt{a^2x\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^5,x)
```

```
[Out] 1/3*a^4*arcsinh(a*x)+1/3*a^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x-1/6*a*arcsinh
(a*x)*(a^2*x^2+1)^(1/2)/x^3-1/12*a^2/x^2-1/4*arcsinh(a*x)^2/x^4-1/3*a^4*ln(
(a*x+(a^2*x^2+1)^(1/2))^2-1)
```

maxima [A] time = 0.31, size = 71, normalized size = 0.84

$$-\frac{1}{12}\left(4\,a^2\log(x)+\frac{1}{x^2}\right)a^2+\frac{1}{6}\left(\frac{2\,\sqrt{a^2x^2+1}\,a^2}{x}-\frac{\sqrt{a^2x^2+1}}{x^3}\right)a\,\operatorname{arsinh}(ax)-\frac{\operatorname{arsinh}(ax)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a^2*log(x) + 1/x^2)*a^2 + 1/6*(2*sqrt(a^2*x^2 + 1)*a^2/x - sqrt(a^
2*x^2 + 1)/x^3)*a*arcsinh(a*x) - 1/4*arcsinh(a*x)^2/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^2/x^5,x)
```

```
[Out] int(asinh(a*x)^2/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**5,x)
```

```
[Out] Integral(asinh(a*x)**2/x**5, x)
```

3.22 $\int x^4 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=195

$$\frac{16x\sinh^{-1}(ax)}{25a^4} - \frac{8x^3\sinh^{-1}(ax)}{75a^2} - \frac{3x^4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{25a} - \frac{6\left(a^2x^2+1\right)^{5/2}}{625a^5} + \frac{76\left(a^2x^2+1\right)^{3/2}}{1125a^5} - \frac{298\sqrt{a^2x^2+1}}{375a^5} -$$

```
[Out] 76/1125*(a^2*x^2+1)^(3/2)/a^5-6/625*(a^2*x^2+1)^(5/2)/a^5+16/25*x*arcsinh(a
*x)/a^4-8/75*x^3*arcsinh(a*x)/a^2+6/125*x^5*arcsinh(a*x)+1/5*x^5*arcsinh(a*
x)^3-298/375*(a^2*x^2+1)^(1/2)/a^5-8/25*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^
5+4/25*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^3-3/25*x^4*arcsinh(a*x)^2*(a^
2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.37, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5661, 5758, 5717, 5653, 261, 266, 43}

$$-\frac{6\left(a^2x^2+1\right)^{5/2}}{625a^5} + \frac{76\left(a^2x^2+1\right)^{3/2}}{1125a^5} - \frac{298\sqrt{a^2x^2+1}}{375a^5} - \frac{3x^4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{25a} - \frac{8x^3\sinh^{-1}(ax)}{75a^2} + \frac{4x^2\sqrt{a^2x^2+1}\ \text{s}}{25a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^3,x]
```

```
[Out] (-298*Sqrt[1 + a^2*x^2])/(375*a^5) + (76*(1 + a^2*x^2)^(3/2))/(1125*a^5) -
(6*(1 + a^2*x^2)^(5/2))/(625*a^5) + (16*x*ArcSinh[a*x])/(25*a^4) - (8*x^3*A
rcSinh[a*x])/(75*a^2) + (6*x^5*ArcSinh[a*x])/125 - (8*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]^2)/(25*a^5) + (4*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(25*a^3) -
 (3*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(25*a) + (x^5*ArcSinh[a*x]^3)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^3\,dx = \frac{1}{5}x^5\sinh^{-1}(ax)^3 - \frac{1}{5}(3a)\int \frac{x^5\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^3 + \frac{6}{25}\int x^4\sinh^{-1}(ax)\,dx + \frac{12\int \frac{x^3\sinh}{\sqrt{1+}}}{25}$$

$$= \frac{6}{125}x^5\sinh^{-1}(ax) + \frac{4x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{25a} + \frac{1}{5}x^5\sinh$$

$$= -\frac{8x^3\sinh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5\sinh^{-1}(ax) - \frac{8\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{25a^5} + \frac{4x^2\sqrt{1+a^2x^2}\,\sinh}{25a^3}$$

$$= \frac{16x\sinh^{-1}(ax)}{25a^4} - \frac{8x^3\sinh^{-1}(ax)}{75a^2} + \frac{6}{125}x^5\sinh^{-1}(ax) - \frac{8\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{25a^5} + \frac{4x}{}$$

$$= -\frac{86\sqrt{1+a^2x^2}}{125a^5} + \frac{4\left(1+a^2x^2\right)^{3/2}}{125a^5} - \frac{6\left(1+a^2x^2\right)^{5/2}}{625a^5} + \frac{16x\sinh^{-1}(ax)}{25a^4} - \frac{8x^3\sinh^{-1}(ax)}{75a^2}$$

$$= -\frac{298\sqrt{1+a^2x^2}}{375a^5} + \frac{76\left(1+a^2x^2\right)^{3/2}}{1125a^5} - \frac{6\left(1+a^2x^2\right)^{5/2}}{625a^5} + \frac{16x\sinh^{-1}(ax)}{25a^4} - \frac{8x^3\sinh^{-1}(a}{75a^2}$$

Mathematica [A] time = 0.08, size = 120, normalized size = 0.62

$$\frac{1125a^5x^5\sinh^{-1}(ax)^3 - 2\sqrt{a^2x^2+1}\left(27a^4x^4 - 136a^2x^2 + 2072\right) + 30ax\left(9a^4x^4 - 20a^2x^2 + 120\right)\sinh^{-1}(ax) -}{5625a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x]^3,x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(2072 - 136*a^2*x^2 + 27*a^4*x^4) + 30*a*x*(120 - 20*
a^2*x^2 + 9*a^4*x^4)*ArcSinh[a*x] - 225*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 +
3*a^4*x^4)*ArcSinh[a*x]^2 + 1125*a^5*x^5*ArcSinh[a*x]^3)/(5625*a^5)
```

fricas [A] time = 0.41, size = 151, normalized size = 0.77

$$\frac{1125\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^3-225\left(3\,a^4x^4-4\,a^2x^2+8\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2+30\left(9\,a^5x^5-20\right.}{5625\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(1125*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^3 - 225*(3*a^4*x^4 - 4*a^
2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 30*(9*a^5*x^5
 - 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(27*a^4*x^4 - 136
*a^2*x^2 + 2072)*sqrt(a^2*x^2 + 1))/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.48, size = 172, normalized size = 0.88

$$\frac{\frac{a^5x^5\operatorname{arcsinh}(ax)^3}{5}-\frac{8\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{25}-\frac{3a^4x^4\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{25}+\frac{4\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,a^2x^2}{25}+\frac{16ax\operatorname{arcsinh}(ax)}{25}-\frac{4144\sqrt{a^2x^2}}{5625}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^3,x)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)^3-8/25*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-3/2
5*a^4*x^4*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+4/25*arcsinh(a*x)^2*(a^2*x^2+1)^
(1/2)*a^2*x^2+16/25*a*x*arcsinh(a*x)-4144/5625*(a^2*x^2+1)^(1/2)+6/125*a^5*
x^5*arcsinh(a*x)-6/625*a^4*x^4*(a^2*x^2+1)^(1/2)+272/5625*a^2*x^2*(a^2*x^2+
1)^(1/2)-8/75*a^3*x^3*arcsinh(a*x))
```

maxima [A] time = 0.35, size = 165, normalized size = 0.85

$$\frac{1}{5}\,x^5\operatorname{arsinh}(ax)^3-\frac{1}{25}\left(\frac{3\sqrt{a^2x^2+1}\,x^4}{a^2}-\frac{4\sqrt{a^2x^2+1}\,x^2}{a^4}+\frac{8\sqrt{a^2x^2+1}}{a^6}\right)a\operatorname{arsinh}(ax)^2-\frac{2}{5625}\,a\left(\frac{27\sqrt{a^2x^2+1}\,a^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^3 - 1/25*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x)^2 - 2/5625*a*((27*s
qrt(a^2*x^2 + 1)*a^2*x^4 - 136*sqrt(a^2*x^2 + 1)*x^2 + 2072*sqrt(a^2*x^2 +
1)/a^2)/a^4 - 15*(9*a^4*x^5 - 20*a^2*x^3 + 120*x)*arcsinh(a*x)/a^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\operatorname{asinh}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asinh(a*x)^3,x)
```

```
[Out] int(x^4*asinh(a*x)^3, x)
```

sympy [A] time = 5.45, size = 196, normalized size = 1.01

$$\begin{cases} \frac{x^5 \operatorname{asinh}^3(ax)}{5} + \frac{6x^5 \operatorname{asinh}(ax)}{125} - \frac{3x^4\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25a} - \frac{6x^4\sqrt{a^2x^2+1}}{625a} - \frac{8x^3\operatorname{asinh}(ax)}{75a^2} + \frac{4x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25a^3} + \frac{272x^2\sqrt{a^2x^2\cdot}}{5625a^3} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**3/5 + 6*x**5*asinh(a*x)/125 - 3*x**4*sqrt(a**2*
x**2 + 1)*asinh(a*x)**2/(25*a) - 6*x**4*sqrt(a**2*x**2 + 1)/(625*a) - 8*x**
3*asinh(a*x)/(75*a**2) + 4*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(25*a**3)
 + 272*x**2*sqrt(a**2*x**2 + 1)/(5625*a**3) + 16*x*asinh(a*x)/(25*a**4) - 8
*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(25*a**5) - 4144*sqrt(a**2*x**2 + 1)/(56
25*a**5), Ne(a, 0)), (0, True))
```

3.23 $\int x^3 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=163

$$-\frac{3\sinh^{-1}(ax)^3}{32a^4}-\frac{45\sinh^{-1}(ax)}{256a^4}-\frac{9x^2\sinh^{-1}(ax)}{32a^2}-\frac{3x^3\sqrt{a^2x^2+1}}{128a}-\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{16a}+\frac{45x\sqrt{a^2x^2+1}}{256a^3}+\ldots$$

```
[Out] -45/256*arcsinh(a*x)/a^4-9/32*x^2*arcsinh(a*x)/a^2+3/32*x^4*arcsinh(a*x)-3/
32*arcsinh(a*x)^3/a^4+1/4*x^4*arcsinh(a*x)^3+45/256*x*(a^2*x^2+1)^(1/2)/a^3
-3/128*x^3*(a^2*x^2+1)^(1/2)/a+9/32*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^3-
3/16*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.30, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5661, 5758, 5675, 321, 215}

$$-\frac{3x^3\sqrt{a^2x^2+1}}{128a}+\frac{45x\sqrt{a^2x^2+1}}{256a^3}-\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{16a}-\frac{9x^2\sinh^{-1}(ax)}{32a^2}+\frac{9x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{32a^3}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^3,x]
```

```
[Out] (45*x*Sqrt[1 + a^2*x^2])/(256*a^3) - (3*x^3*Sqrt[1 + a^2*x^2])/(128*a) - (4
5*ArcSinh[a*x])/(256*a^4) - (9*x^2*ArcSinh[a*x])/(32*a^2) + (3*x^4*ArcSinh[
a*x])/32 + (9*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^2)/(16*a) - (3*ArcSinh[a*x]^3)/(32*a^4) + (x^4*ArcS
inh[a*x]^3)/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
```

```
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^3\,dx = \frac{1}{4}x^4\sinh^{-1}(ax)^3 - \frac{1}{4}(3a)\int \frac{x^4\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{16a} + \frac{1}{4}x^4\sinh^{-1}(ax)^3 + \frac{3}{8}\int x^3\sinh^{-1}(ax)\,dx + \frac{9\int \frac{x^2\sinh^{-1}}{\sqrt{1+a^2}}}{16a}$$

$$= \frac{3}{32}x^4\sinh^{-1}(ax) + \frac{9x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{32a^3} - \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{16a} + \frac{1}{4}x^4\sinh$$

$$= -\frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{9x^2\sinh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\sinh^{-1}(ax) + \frac{9x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{32a^3} - 3$$

$$= \frac{45x\sqrt{1+a^2x^2}}{256a^3} - \frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{9x^2\sinh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\sinh^{-1}(ax) + \frac{9x\sqrt{1+a^2x^2}}{32a}$$

$$= \frac{45x\sqrt{1+a^2x^2}}{256a^3} - \frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{45\sinh^{-1}(ax)}{256a^4} - \frac{9x^2\sinh^{-1}(ax)}{32a^2} + \frac{3}{32}x^4\sinh^{-1}(ax)$$

Mathematica [A] time = 0.07, size = 110, normalized size = 0.67

$$\frac{8\left(8a^4x^4-3\right)\sinh^{-1}(ax)^3 + 3ax\left(15-2a^2x^2\right)\sqrt{a^2x^2+1} - 24ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)^2 + 3\left(8a^4x^4\right.}{256a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x]^3,x]
```

```
[Out] (3*a*x*(15 - 2*a^2*x^2)*Sqrt[1 + a^2*x^2] + 3*(-15 - 24*a^2*x^2 + 8*a^4*x^4
)*ArcSinh[a*x] - 24*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2 +
 8*(-3 + 8*a^4*x^4)*ArcSinh[a*x]^3)/(256*a^4)
```

fricas [A] time = 0.45, size = 142, normalized size = 0.87

$$\frac{8\left(8a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 24\left(2a^3x^3-3ax\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 3\left(8a^4x^4-24a\right.}{256a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 24*(2*a^3*x^3 - 3
*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 3*(8*a^4*x^4 - 24*
a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1)) - 3*(2*a^3*x^3 - 15*a*x)*sqrt(a^
2*x^2 + 1))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.38, size = 141, normalized size = 0.87

$$\frac{\frac{a^4x^4\operatorname{arcsinh}(ax)^3}{4}-\frac{3a^3x^3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{16}+\frac{9ax\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{32}-\frac{3\operatorname{arcsinh}(ax)^3}{32}+\frac{3\operatorname{arcsinh}(ax)a^4x^4}{32}-\frac{3a^3x^3\sqrt{a^2x^2+1}}{128}+\frac{45}{}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^3,x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)^3-3/16*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(
1/2)+9/32*a*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-3/32*arcsinh(a*x)^3+3/32*arc
sinh(a*x)*a^4*x^4-3/128*a^3*x^3*(a^2*x^2+1)^(1/2)+45/256*a*x*(a^2*x^2+1)^(1
/2)+27/256*arcsinh(a*x)-9/32*(a^2*x^2+1)*arcsinh(a*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^4\log\left(ax+\sqrt{a^2x^2+1}\right)^3-\int\frac{3\left(a^3x^6+\sqrt{a^2x^2+1}\,a^2x^5+ax^4\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\left(a^3x^3+ax+\left(a^2x^2+1\right)^{\frac{3}{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1))^3 - integrate(3/4*(a^3*x^6 + sqrt(a^2*
x^2 + 1)*a^2*x^5 + a*x^4)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\operatorname{asinh}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x)^3,x)
```

```
[Out] int(x^3*asinh(a*x)^3, x)
```

sympy [A] time = 3.15, size = 160, normalized size = 0.98

$$\begin{cases}\frac{x^4\operatorname{asinh}^3(ax)}{4}+\frac{3x^4\operatorname{asinh}(ax)}{32}-\frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{16a}-\frac{3x^3\sqrt{a^2x^2+1}}{128a}-\frac{9x^2\operatorname{asinh}(ax)}{32a^2}+\frac{9x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{32a^3}+\frac{45x\sqrt{a^2x^2+1}}{256a^3}- \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**3/4 + 3*x**4*asinh(a*x)/32 - 3*x**3*sqrt(a**2*x
**2 + 1)*asinh(a*x)**2/(16*a) - 3*x**3*sqrt(a**2*x**2 + 1)/(128*a) - 9*x**2
*asinh(a*x)/(32*a**2) + 9*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(32*a**3) + 4
5*x*sqrt(a**2*x**2 + 1)/(256*a**3) - 3*asinh(a*x)**3/(32*a**4) - 45*asinh(a
*x)/(256*a**4), Ne(a, 0)), (0, True))
```

3.24 $\int x^2 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=132

$$-\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a}-\frac{4x\sinh^{-1}(ax)}{3a^2}-\frac{2\left(a^2x^2+1\right)^{3/2}}{27a^3}+\frac{14\sqrt{a^2x^2+1}}{9a^3}+\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\sinh$$

```
[Out] -2/27*(a^2*x^2+1)^(3/2)/a^3-4/3*x*arcsinh(a*x)/a^2+2/9*x^3*arcsinh(a*x)+1/3
*x^3*arcsinh(a*x)^3+14/9*(a^2*x^2+1)^(1/2)/a^3+2/3*arcsinh(a*x)^2*(a^2*x^2+
1)^(1/2)/a^3-1/3*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.22, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5661, 5758, 5717, 5653, 261, 266, 43}

$$-\frac{2\left(a^2x^2+1\right)^{3/2}}{27a^3}+\frac{14\sqrt{a^2x^2+1}}{9a^3}-\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a}+\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a^3}-\frac{4x\sinh^{-1}(ax)}{3a^2}+\frac{1}{3}x^3\sinh$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^3,x]
```

```
[Out] (14*Sqrt[1 + a^2*x^2])/(9*a^3) - (2*(1 + a^2*x^2)^(3/2))/(27*a^3) - (4*x*Ar
cSinh[a*x])/(3*a^2) + (2*x^3*ArcSinh[a*x])/9 + (2*Sqrt[1 + a^2*x^2]*ArcSinh
[a*x]^2)/(3*a^3) - (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a) + (x^3*ArcS
inh[a*x]^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \sinh^{-1}(ax)^3\,dx = \frac{1}{3}x^3\sinh^{-1}(ax)^3 - a\int \frac{x^3\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a} + \frac{1}{3}x^3\sinh^{-1}(ax)^3 + \frac{2}{3}\int x^2\sinh^{-1}(ax)\,dx + \frac{2\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,d}{3a}$$
$$= \frac{2}{9}x^3\sinh^{-1}(ax) + \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a} + \frac{1}{3}x^3\sinh^{-1}(ax)$$
$$= -\frac{4x\sinh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3\sinh^{-1}(ax) + \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a}$$
$$= \frac{4\sqrt{1+a^2x^2}}{3a^3} - \frac{4x\sinh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3\sinh^{-1}(ax) + \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x}}{}$$
$$= \frac{14\sqrt{1+a^2x^2}}{9a^3} - \frac{2\left(1+a^2x^2\right)^{3/2}}{27a^3} - \frac{4x\sinh^{-1}(ax)}{3a^2} + \frac{2}{9}x^3\sinh^{-1}(ax) + \frac{2\sqrt{1+a^2x^2}\,\sinh^{-1}(}{3a^3}$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.70

$$\frac{9a^3x^3\sinh^{-1}(ax)^3 - 2\left(a^2x^2-20\right)\sqrt{a^2x^2+1} - 9\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2 + 6ax\left(a^2x^2-6\right)\sinh^{-1}(ax)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x]^3,x]
```

```
[Out] (-2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] + 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x] -
 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 + 9*a^3*x^3*ArcSinh[a*x]
^3)/(27*a^3)
```

fricas [A] time = 0.42, size = 124, normalized size = 0.94

$$\frac{9\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 9\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 6\left(a^3x^3-6\,ax\right)\log\left(ax+\sqrt{a^2x^2}\right.}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*x
^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*x^3 - 6*a*x)*log(a*x + sqrt
(a^2*x^2 + 1)) - 2*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20))/a^3
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.38, size = 116, normalized size = 0.88

$$\frac{\frac{a^3x^3\operatorname{arcsinh}(ax)^3}{3}+\frac{2\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{3}-\frac{\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,a^2x^2}{3}-\frac{4ax\operatorname{arcsinh}(ax)}{3}+\frac{40\sqrt{a^2x^2+1}}{27}+\frac{2a^3x^3\operatorname{arcsinh}(ax)}{9}-\frac{2a^2x}{}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)^3+2/3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1/3*
arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^2*x^2-4/3*a*x*arcsinh(a*x)+40/27*(a^2*x^
2+1)^(1/2)+2/9*a^3*x^3*arcsinh(a*x)-2/27*a^2*x^2*(a^2*x^2+1)^(1/2))
```

maxima [**A**] time = 0.33, size = 116, normalized size = 0.88

$$\frac{1}{3}x^3\operatorname{arsinh}(ax)^3-\frac{1}{3}a\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^2-\frac{2}{27}a\left(\frac{\sqrt{a^2x^2+1}\,x^2-\frac{20\sqrt{a^2x^2+1}}{a^2}}{a^2}-\frac{3\left(a^2x^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^3 - 1/3*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x)^2 - 2/27*a*((sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2
 + 1)/a^2)/a^2 - 3*(a^2*x^3 - 6*x)*arcsinh(a*x)/a^3)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\operatorname{asinh}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x)^3,x)
```

```
[Out] int(x^2*asinh(a*x)^3, x)
```

sympy [**A**] time = 1.80, size = 128, normalized size = 0.97

$$\begin{cases}\frac{x^3\operatorname{asinh}^3(ax)}{3}+\frac{2x^3\operatorname{asinh}(ax)}{9}-\frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a}-\frac{2x^2\sqrt{a^2x^2+1}}{27a}-\frac{4x\operatorname{asinh}(ax)}{3a^2}+\frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^3}+\frac{40\sqrt{a^2x^2+1}}{27a^3} & \text{fo} \\ 0 & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**3/3 + 2*x**3*asinh(a*x)/9 - x**2*sqrt(a**2*x**2
 + 1)*asinh(a*x)**2/(3*a) - 2*x**2*sqrt(a**2*x**2 + 1)/(27*a) - 4*x*asinh(a
*x)/(3*a**2) + 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**3) + 40*sqrt(a**2*
x**2 + 1)/(27*a**3), Ne(a, 0)), (0, True))
```

3.25 $\int x \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=97

$$-\frac{3x\sqrt{a^2x^2+1}}{8a}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{4a}+\frac{\sinh^{-1}(ax)^3}{4a^2}+\frac{3\sinh^{-1}(ax)}{8a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^3+\frac{3}{4}x^2\sinh^{-1}(ax)$$

```
[Out] 3/8*arcsinh(a*x)/a^2+3/4*x^2*arcsinh(a*x)+1/4*arcsinh(a*x)^3/a^2+1/2*x^2*ar
csinh(a*x)^3-3/8*x*(a^2*x^2+1)^(1/2)/a-3/4*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/
2)/a
```

Rubi [A] time = 0.15, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5661, 5758, 5675, 321, 215}

$$-\frac{3x\sqrt{a^2x^2+1}}{8a}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{4a}+\frac{\sinh^{-1}(ax)^3}{4a^2}+\frac{3\sinh^{-1}(ax)}{8a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^3+\frac{3}{4}x^2\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*x*Sqrt[1 + a^2*x^2])/(8*a) + (3*ArcSinh[a*x])/(8*a^2) + (3*x^2*ArcSinh[
a*x])/4 - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*a) + ArcSinh[a*x]^3/(4*
a^2) + (x^2*ArcSinh[a*x]^3)/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x\sinh^{-1}(ax)^3\,dx = \frac{1}{2}x^2\sinh^{-1}(ax)^3 - \frac{1}{2}(3a)\int\frac{x^2\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4a} + \frac{1}{2}x^2\sinh^{-1}(ax)^3 + \frac{3}{2}\int x\sinh^{-1}(ax)\,dx + \frac{3\int\frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a}$$
$$= \frac{3}{4}x^2\sinh^{-1}(ax) - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4a} + \frac{\sinh^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^3 - \frac{1}{4}(3a)\int$$
$$= -\frac{3x\sqrt{1+a^2x^2}}{8a} + \frac{3}{4}x^2\sinh^{-1}(ax) - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4a} + \frac{\sinh^{-1}(ax)^3}{4a^2} + \frac{1}{2}x^2\sinh^{-1}$$
$$= -\frac{3x\sqrt{1+a^2x^2}}{8a} + \frac{3\sinh^{-1}(ax)}{8a^2} + \frac{3}{4}x^2\sinh^{-1}(ax) - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4a} + \frac{\sinh^{-1}(ax}{4a^2}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.82

$$\frac{-3ax\sqrt{a^2x^2+1} + \left(4a^2x^2+2\right)\sinh^{-1}(ax)^3 - 6ax\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2 + \left(6a^2x^2+3\right)\sinh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*a*x*Sqrt[1 + a^2*x^2] + (3 + 6*a^2*x^2)*ArcSinh[a*x] - 6*a*x*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^2 + (2 + 4*a^2*x^2)*ArcSinh[a*x]^3)/(8*a^2)
```

fricas [A] time = 0.41, size = 112, normalized size = 1.15

$$-\frac{6\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 2\left(2a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 3\sqrt{a^2x^2+1}\,ax - 3\left(2a^2x^2+1\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*(2*a^2*x^2
 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2*x
^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.08, size = 88, normalized size = 0.91

$$\frac{\frac{\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^3}{2} - \frac{3ax\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{4} - \frac{\operatorname{arcsinh}(ax)^3}{4} + \frac{3\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)}{4} - \frac{3ax\sqrt{a^2x^2+1}}{8} - \frac{3\operatorname{arcsinh}(ax)}{8}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^3,x)
```

```
[Out] 1/a^2*(1/2*(a^2*x^2+1)*arcsinh(a*x)^3-3/4*a*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1
/2)-1/4*arcsinh(a*x)^3+3/4*(a^2*x^2+1)*arcsinh(a*x)-3/8*a*x*(a^2*x^2+1)^(1/
2)-3/8*arcsinh(a*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2\log\left(ax+\sqrt{a^2x^2+1}\right)^3-\int\frac{3\left(a^3x^4+\sqrt{a^2x^2+1}\,a^2x^3+ax^2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\left(a^3x^3+ax+\left(a^2x^2+1\right)^{\frac{3}{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1))^3 - integrate(3/2*(a^3*x^4 + sqrt(a^2*
x^2 + 1)*a^2*x^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{asinh}\left(a\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x)^3,x)
```

```
[Out] int(x*asinh(a*x)^3, x)
```

sympy [A] time = 0.87, size = 92, normalized size = 0.95

$$\begin{cases}\frac{x^2\operatorname{asinh}^3(ax)}{2}+\frac{3x^2\operatorname{asinh}(ax)}{4}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a}-\frac{3x\sqrt{a^2x^2+1}}{8a}+\frac{\operatorname{asinh}^3(ax)}{4a^2}+\frac{3\operatorname{asinh}(ax)}{8a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**3/2 + 3*x**2*asinh(a*x)/4 - 3*x*sqrt(a**2*x**2
+ 1)*asinh(a*x)**2/(4*a) - 3*x*sqrt(a**2*x**2 + 1)/(8*a) + asinh(a*x)**3/(4
*a**2) + 3*asinh(a*x)/(8*a**2), Ne(a, 0)), (0, True))
```

3.26 $\int \sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=58

$$-\frac{6\sqrt{a^2x^2+1}}{a}-\frac{3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3+6x\sinh^{-1}(ax)$$

```
[Out] 6*x*arcsinh(a*x)+x*arcsinh(a*x)^3-6*(a^2*x^2+1)^(1/2)/a-3*arcsinh(a*x)^2*(a
^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5653, 5717, 261}

$$-\frac{6\sqrt{a^2x^2+1}}{a}-\frac{3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3+6x\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + a^2*x^2])/a + 6*x*ArcSinh[a*x] - (3*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^2)/a + x*ArcSinh[a*x]^3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}(ax)^3\,dx &= x\sinh^{-1}(ax)^3-(3a)\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3+6\int \sinh^{-1}(ax)\,dx\\ &= 6x\sinh^{-1}(ax)-\frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3-(6a)\int \frac{x}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{6\sqrt{1+a^2x^2}}{a}+6x\sinh^{-1}(ax)-\frac{3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.00

$$-\frac{6\sqrt{a^2x^2+1}}{a}-\frac{3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{a}+x\sinh^{-1}(ax)^3+6x\sinh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + a^2*x^2])/a + 6*x*ArcSinh[a*x] - (3*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^2)/a + x*ArcSinh[a*x]^3
```

fricas [A] time = 0.40, size = 90, normalized size = 1.55

$$\frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3+6\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2-6\,\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 + 6*a*x*log(a*x + sqrt(a^2*x^2 + 1)) -
3*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^2*x^2 + 1))/a
```

giac [A] time = 0.18, size = 98, normalized size = 1.69

$$x\log\left(ax+\sqrt{a^2x^2+1}\right)^3-3\,a\left(\frac{\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^2}-\frac{2\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a*(sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^
2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a
)/a)
```

maple [A] time = 0.10, size = 55, normalized size = 0.95

$$\frac{ax\,\mathrm{arcsinh}\,(ax)^3-3\,\mathrm{arcsinh}\,(ax)^2\,\sqrt{a^2x^2+1}\,+6ax\,\mathrm{arcsinh}\,(ax)-6\sqrt{a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3,x)
```

```
[Out] 1/a*(a*x*arcsinh(a*x)^3-3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+6*a*x*arcsinh(a*
x)-6*(a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.31, size = 57, normalized size = 0.98

$$x\,\mathrm{arsinh}\,(ax)^3-\frac{3\,\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)^2}{a}+\frac{6\left(ax\,\mathrm{arsinh}\,(ax)-\sqrt{a^2x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^3 - 3*sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a + 6*(a*x*arcsinh(a*
x) - sqrt(a^2*x^2 + 1))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asinh}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3,x)
```

```
[Out] int(asinh(a*x)^3, x)
```

sympy [A] time = 0.43, size = 54, normalized size = 0.93

$$\begin{cases} x\operatorname{asinh}^3(ax) + 6x\operatorname{asinh}(ax) - \frac{3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{a} - \frac{6\sqrt{a^2x^2+1}}{a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3,x)
```

```
[Out] Piecewise((x*asinh(a*x)**3 + 6*x*asinh(a*x) - 3*sqrt(a**2*x**2 + 1)*asinh(a
*x)**2/a - 6*sqrt(a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))
```

3.27 $\int \frac{\sinh^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=83

$$\frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}\sinh^{-1}(ax)\text{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)+\frac{3}{4}\text{Li}_4\left(e^{2\sinh^{-1}(ax)}\right)-\frac{1}{4}\sinh^{-1}(ax)^4+\sinh^{-1}(ax)^3$$

```
[Out] -1/4*arcsinh(a*x)^4+arcsinh(a*x)^3*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2*arcs
inh(a*x)^2*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*arcsinh(a*x)*polylog(3,
(a*x+(a^2*x^2+1)^(1/2))^2)+3/4*polylog(4,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}\sinh^{-1}(ax)\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)+\frac{3}{4}\text{PolyLog}\left(4,e^{2\sinh^{-1}(ax)}\right)-\frac{1}{4}\text{s}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x,x]
```

```
[Out] -ArcSinh[a*x]^4/4 + ArcSinh[a*x]^3*Log[1 - E^(2*ArcSinh[a*x])] + (3*ArcSinh
[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2 - (3*ArcSinh[a*x]*PolyLog[3, E^(2
*ArcSinh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcSinh[a*x])])/4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^3}{x}\,dx &= \text{Subst}\left(\int x^3\coth(x)\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{4}\sinh^{-1}(ax)^4 - 2\,\text{Subst}\left(\int \frac{e^{2x}x^3}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) - 3\,\text{Subst}\left(\int x^2\log\left(1-e^{2x}\right)\,dx, x, \text{sinh}\right.\\
&= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\,\text{Sul}\\
&= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{3}{2}\,\text{sir}\\
&= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{3}{2}\,\text{sir}\\
&= -\frac{1}{4}\sinh^{-1}(ax)^4 + \sinh^{-1}(ax)^3\log\left(1-e^{2\sinh^{-1}(ax)}\right) + \frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - \frac{3}{2}\,\text{sir}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 83, normalized size = 1.00

$$
\frac{3}{2}\sinh^{-1}(ax)^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}\sinh^{-1}(ax)\text{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)+\frac{3}{4}\text{Li}_4\left(e^{2\sinh^{-1}(ax)}\right)-\frac{1}{4}\sinh^{-1}(ax)^4+\sinh^{-1}(ax)^3\,\text{log}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/x,x]
```

```
[Out] -1/4*ArcSinh[a*x]^4 + ArcSinh[a*x]^3*Log[1 - E^(2*ArcSinh[a*x])] + (3*ArcSi
nh[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2 - (3*ArcSinh[a*x]*PolyLog[3, E^
(2*ArcSinh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcSinh[a*x])])/4
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\text{arsinh}\,(ax)^3}{x}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x, x)
```

maple [A] time = 0.16, size = 204, normalized size = 2.46

$$-\frac{\operatorname{arcsinh}(ax)^4}{4}+\operatorname{arcsinh}(ax)^3\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+3\operatorname{arcsinh}(ax)^2\operatorname{polylog}\left(2, ax+\sqrt{a^2x^2+1}\right)-6\operatorname{arcsi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x,x)
```

```
[Out] -1/4*arcsinh(a*x)^4+arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*arcsinh(a*
x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2
+1)^(1/2))+6*polylog(4,a*x+(a^2*x^2+1)^(1/2))+arcsinh(a*x)^3*ln(a*x+(a^2*x^
2+1)^(1/2)+1)+3*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-6*arcsinh(
a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+6*polylog(4,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/x,x)
```

```
[Out] int(asinh(a*x)^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x,x)
```

```
[Out] Integral(asinh(a*x)**3/x, x)
```

3.28 $\int \frac{\sinh^{-1}(ax)^3}{x^2} dx$

Optimal. Leaf size=84

$$-6a\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+6a\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)+6a\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)-6a\text{Li}_3\left(e^{\sinh^{-1}(ax)}\right)-\frac{\sinh^{-1}(a}{x}$$

```
[Out] -arcsinh(a*x)^3/x-6*a*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-6*a*arc
sinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,a*x+
(a^2*x^2+1)^(1/2))+6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*a*polylog(3,a*x+
(a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.16, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5661, 5760, 4182, 2531, 2282, 6589}

$$-6a\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+6a\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+6a\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)-6a\text{P}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x^2,x]
```

```
[Out] -(ArcSinh[a*x]^3/x) - 6*a*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 6*a*ArcS
inh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] + 6*a*ArcSinh[a*x]*PolyLog[2, E^ArcSin
h[a*x]] + 6*a*PolyLog[3, -E^ArcSinh[a*x]] - 6*a*PolyLog[3, E^ArcSinh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^3}{x^2}\,dx &= -\frac{\sinh^{-1}(ax)^3}{x} + (3a)\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\sinh^{-1}(ax)^3}{x} + (3a)\,\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right) \\
&= -\frac{\sinh^{-1}(ax)^3}{x} - 6a\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - (6a)\,\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, s\right. \\
&= -\frac{\sinh^{-1}(ax)^3}{x} - 6a\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 6a\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 6a \\
&= -\frac{\sinh^{-1}(ax)^3}{x} - 6a\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 6a\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 6a \\
&= -\frac{\sinh^{-1}(ax)^3}{x} - 6a\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 6a\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 6a
\end{aligned}$$

Mathematica [A] time = 0.13, size = 117, normalized size = 1.39

$$a\left(6\sinh^{-1}(ax)\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) - 6\sinh^{-1}(ax)\text{Li}_2\left(e^{-\sinh^{-1}(ax)}\right) + 6\text{Li}_3\left(-e^{-\sinh^{-1}(ax)}\right) - 6\text{Li}_3\left(e^{-\sinh^{-1}(ax)}\right) -\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/x^2,x]
```

```
[Out] a*(-(ArcSinh[a*x]^3/(a*x)) + 3*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] -
3*ArcSinh[a*x]^2*Log[1 + E^(-ArcSinh[a*x])] + 6*ArcSinh[a*x]*PolyLog[2, -E^
(-ArcSinh[a*x])] - 6*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x])] + 6*PolyLog
[3, -E^(-ArcSinh[a*x])] - 6*PolyLog[3, E^(-ArcSinh[a*x])])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x^2, x)
```

maple [A] time = 0.23, size = 162, normalized size = 1.93

$$-\frac{\operatorname{arcsinh}(ax)^3}{x}+3a\operatorname{arcsinh}(ax)^2\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+6a\operatorname{arcsinh}(ax)\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2+1}\right)-6a\operatorname{pol}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^2,x)
```

```
[Out] -arcsinh(a*x)^3/x+3*a*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+6*a*arcsin
h(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*a*polylog(3,a*x+(a^2*x^2+1)^(1/2)
)-3*a*arcsinh(a*x)^2*ln(a*x+(a^2*x^2+1)^(1/2)+1)-6*a*arcsinh(a*x)*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))+6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{x}+\int\frac{3\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^3x^4+ax^2+\left(a^2x^3+x\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^4 + a*x^2 + (a^2*x^3 + x
)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{asinh}(a\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/x^2,x)
```

```
[Out] int(asinh(a*x)^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}^3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**2,x)
```

```
[Out] Integral(asinh(a*x)**3/x**2, x)
```

3.29 $\int \frac{\sinh^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=93

$$\frac{3}{2}a^2\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-\frac{3a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2x}-\frac{3}{2}a^2\sinh^{-1}(ax)^2+3a^2\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)-\frac{\sinh^{-1}}{2}$$

[Out] -3/2*a^2*arcsinh(a*x)^2-1/2*arcsinh(a*x)^3/x^2+3*a^2*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2*a^2*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x

Rubi [A] time = 0.17, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5661, 5723, 5659, 3716, 2190, 2279, 2391}

$$\frac{3}{2}a^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2x}-\frac{3}{2}a^2\sinh^{-1}(ax)^2+3a^2\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x^3,x]
```

```
[Out] (-3*a^2*ArcSinh[a*x]^2)/2 - (3*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x) -
ArcSinh[a*x]^3/(2*x^2) + 3*a^2*ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + (
3*a^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^3}{x^3}\,dx &= -\frac{\sinh^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{3a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x}-\frac{\sinh^{-1}(ax)^3}{2x^2}+\left(3a^2\right)\int \frac{\sinh^{-1}(ax)}{x}\,dx\\
&= -\frac{3a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x}-\frac{\sinh^{-1}(ax)^3}{2x^2}+\left(3a^2\right)\text{Subst}\left(\int x\coth(x)\,dx,x,\sinh^{-1}(ax)\right)\\
&= -\frac{3}{2}a^2\sinh^{-1}(ax)^2-\frac{3a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x}-\frac{\sinh^{-1}(ax)^3}{2x^2}-\left(6a^2\right)\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx,\right.\\
&= -\frac{3}{2}a^2\sinh^{-1}(ax)^2-\frac{3a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x}-\frac{\sinh^{-1}(ax)^3}{2x^2}+3a^2\sinh^{-1}(ax)\log\left(1-e^{2\,\text{si}}\right.\\
&= -\frac{3}{2}a^2\sinh^{-1}(ax)^2-\frac{3a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x}-\frac{\sinh^{-1}(ax)^3}{2x^2}+3a^2\sinh^{-1}(ax)\log\left(1-e^{2\,\text{si}}\right.\\
&= -\frac{3}{2}a^2\sinh^{-1}(ax)^2-\frac{3a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x}-\frac{\sinh^{-1}(ax)^3}{2x^2}+3a^2\sinh^{-1}(ax)\log\left(1-e^{2\,\text{si}}\right.
\end{aligned}$$

Mathematica [A] time = 0.35, size = 80, normalized size = 0.86

$$-\frac{\sinh^{-1}(ax)^3-3ax\left(\sinh^{-1}(ax)\left(\left(ax-\sqrt{a^2x^2+1}\right)\sinh^{-1}(ax)+2ax\log\left(1-e^{-2\sinh^{-1}(ax)}\right)\right)-ax\text{Li}_2\left(e^{-2\sinh^{-1}(ax}\right.\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/x^3,x]
```

```
[Out] -1/2*(ArcSinh[a*x]^3 - 3*a*x*(ArcSinh[a*x]*((a*x - Sqrt[1 + a^2*x^2])*ArcSi
nh[a*x] + 2*a*x*Log[1 - E^(-2*ArcSinh[a*x])]) - a*x*PolyLog[2, E^(-2*ArcSin
h[a*x])]))/x^2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\left(ax\right)^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.37, size = 149, normalized size = 1.60

$$-\frac{3a^2 \operatorname{arcsinh}(ax)^2}{2}-\frac{3a \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1}}{2x}-\frac{\operatorname{arcsinh}(ax)^3}{2x^2}+3a^2 \operatorname{arcsinh}(ax) \ln\left(1-ax-\sqrt{a^2x^2+1}\right)+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^3,x)
```

```
[Out] -3/2*a^2*arcsinh(a*x)^2-3/2*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x-1/2*arcsin
h(a*x)^3/x^2+3*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(2
,a*x+(a^2*x^2+1)^(1/2))+3*a^2*arcsinh(a*x)*ln(a*x+(a^2*x^2+1)^(1/2)+1)+3*a^
2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{2\,x^2}+\int \frac{3\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\left(a^3x^5+ax^3+\left(a^2x^4+x^2\right)\sqrt{a^2x^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + sqrt(a^2*x^2 + 1))^3/x^2 + integrate(3/2*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^5 + a*x^3 + (a^2
*x^4 + x^2)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/x^3,x)
```

```
[Out] int(asinh(a*x)^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**3,x)
```

```
[Out] Integral(asinh(a*x)**3/x**3, x)
```

3.30 $\int \frac{\sinh^{-1}(ax)^3}{x^4}\,dx$

Optimal. Leaf size=151

$$a^3\sinh^{-1}(ax)\mathrm{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-a^3\sinh^{-1}(ax)\mathrm{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-a^3\mathrm{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)+a^3\mathrm{Li}_3\left(e^{\sinh^{-1}(ax)}\right)+a^3\sinh^{-1}(a$$

```
[Out] -a^2*arcsinh(a*x)/x-1/3*arcsinh(a*x)^3/x^3+a^3*arcsinh(a*x)^2*arctanh(a*x+(
a^2*x^2+1)^(1/2))-a^3*arctanh((a^2*x^2+1)^(1/2))+a^3*arcsinh(a*x)*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))-a^3*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-
a^3*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+a^3*polylog(3,a*x+(a^2*x^2+1)^(1/2))-
1/2*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.28, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5661, 5747, 5760, 4182, 2531, 2282, 6589, 266, 63, 208}

$$a^3\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-a^3\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-a^3\mathrm{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)+a^3\mathrm{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x^4,x]
```

```
[Out] -((a^2*ArcSinh[a*x])/x) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) - Ar
cSinh[a*x]^3/(3*x^3) + a^3*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^3*Arc
Tanh[Sqrt[1 + a^2*x^2]] + a^3*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^
3*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] - a^3*PolyLog[3, -E^ArcSinh[a*x]]
 + a^3*PolyLog[3, E^ArcSinh[a*x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x^4}\,dx = -\frac{\sinh^{-1}(ax)^3}{3x^3} + a\int \frac{\sinh^{-1}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^2\int \frac{\sinh^{-1}(ax)}{x^2}\,dx - \frac{1}{2}a^3\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} - \frac{1}{2}a^3\,\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x,\right.$$

$$= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^3\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(a}\right.$$

$$= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^3\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(a}\right.$$

$$= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^3\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(a}\right.$$

$$= -\frac{a^2\sinh^{-1}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x^2} - \frac{\sinh^{-1}(ax)^3}{3x^3} + a^3\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(a}\right.$$

Mathematica [A] time = 2.25, size = 268, normalized size = 1.77

$$\frac{1}{48}a^3\left(-\frac{16\sinh^{-1}(ax)^3\sinh^4\left(\frac{1}{2}\sinh^{-1}(ax)\right)}{a^3x^3} - 48\sinh^{-1}(ax)\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) + 48\sinh^{-1}(ax)\text{Li}_2\left(e^{-\sinh^{-1}(ax)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/x^4,x]
```

```
[Out] (a^3*(-24*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] + 4*ArcSinh[a*x]^3*Coth[ArcSinh
[a*x]/2] - 6*ArcSinh[a*x]^2*Csch[ArcSinh[a*x]/2]^2 - a*x*ArcSinh[a*x]^3*Csc
h[ArcSinh[a*x]/2]^4 - 24*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 24*Arc
Sinh[a*x]^2*Log[1 + E^(-ArcSinh[a*x])] + 48*Log[Tanh[ArcSinh[a*x]/2]] - 48*
ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] + 48*ArcSinh[a*x]*PolyLog[2, E^
(-ArcSinh[a*x])] - 48*PolyLog[3, -E^(-ArcSinh[a*x])] + 48*PolyLog[3, E^(-Ar
cSinh[a*x])] - 6*ArcSinh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 - (16*ArcSinh[a*x]^3
*Sinh[ArcSinh[a*x]/2]^4)/(a^3*x^3) + 24*ArcSinh[a*x]*Tanh[ArcSinh[a*x]/2] -
 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2]))/48
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x^4, x)
```

maple [A] time = 0.54, size = 228, normalized size = 1.51

$$-\frac{a \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1}}{2x^2}-\frac{a^2 \operatorname{arcsinh}(ax)}{x}-\frac{\operatorname{arcsinh}(ax)^3}{3x^3}-\frac{a^3 \operatorname{arcsinh}(ax)^2 \ln\left(1-ax-\sqrt{a^2x^2+1}\right)}{2}-a^3 \mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^4,x)
```

```
[Out] -1/2*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x^2-a^2*arcsinh(a*x)/x-1/3*arcsinh(
a*x)^3/x^3-1/2*a^3*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))-a^3*arcsinh(a
*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+a^3*polylog(3,a*x+(a^2*x^2+1)^(1/2))+1
/2*a^3*arcsinh(a*x)^2*ln(a*x+(a^2*x^2+1)^(1/2)+1)+a^3*arcsinh(a*x)*polylog(
2,-a*x-(a^2*x^2+1)^(1/2))-a^3*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-2*a^3*arcta
nh(a*x+(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{3\,x^3}+\int \frac{\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^3x^6+ax^4+\left(a^2x^5+x^3\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/x^3 + integrate((a^3*x^2 + sqrt(a^2*x^2
 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^6 + a*x^4 + (a^2*x^5
 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/x^4,x)
```

```
[Out] int(asinh(a*x)^3/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**4,x)
```

```
[Out] Integral(asinh(a*x)**3/x**4, x)
```

3.31 $\int \frac{\sinh^{-1}(ax)^3}{x^5}\,dx$

Optimal. Leaf size=159

$$-\frac{1}{2}a^4\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)+\frac{1}{2}a^4\sinh^{-1}(ax)^2-a^4\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)-\frac{a^2\sinh^{-1}(ax)}{4x^2}-\frac{a\sqrt{a^2x^2+1}\,\sinh^{-1}(}{4x^3}$$

```
[Out] -1/4*a^2*arcsinh(a*x)/x^2+1/2*a^4*arcsinh(a*x)^2-1/4*arcsinh(a*x)^3/x^4-a^4
*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)-1/2*a^4*polylog(2,(a*x+(a^2*x
^2+1)^(1/2))^2)-1/4*a^3*(a^2*x^2+1)^(1/2)/x-1/4*a*arcsinh(a*x)^2*(a^2*x^2+1
)^(1/2)/x^3+1/2*a^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.29, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5661, 5747, 5723, 5659, 3716, 2190, 2279, 2391, 264}

$$-\frac{1}{2}a^4\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{a^3\sqrt{a^2x^2+1}}{4x}+\frac{a^3\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{2x}-\frac{a^2\sinh^{-1}(ax)}{4x^2}-\frac{a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/x^5,x]
```

```
[Out] -(a^3*Sqrt[1 + a^2*x^2])/(4*x) - (a^2*ArcSinh[a*x])/(4*x^2) + (a^4*ArcSinh[
a*x]^2)/2 - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*x^3) + (a^3*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^2)/(2*x) - ArcSinh[a*x]^3/(4*x^4) - a^4*ArcSinh[a*x]*Lo
g[1 - E^(2*ArcSinh[a*x])] - (a^4*PolyLog[2, E^(2*ArcSinh[a*x])])/2
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
```

```
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^3}{x^5}\,dx &= -\frac{\sinh^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a)\int \frac{\sinh^{-1}(ax)^2}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4x^3} - \frac{\sinh^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)}{x^3}\,dx - \frac{1}{2}a^3\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a^2\sinh^{-1}(ax)}{4x^2} - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x} - \frac{\sinh^{-1}(ax)^3}{4x^4} + \frac{1}{4} \\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{2x} - \\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\sinh^{-1}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4x^3} + \\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\sinh^{-1}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4x^3} + \\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\sinh^{-1}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4x^3} + \\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\sinh^{-1}(ax)}{4x^2} + \frac{1}{2}a^4\sinh^{-1}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}{4x^3} +
\end{aligned}
$$

Mathematica [A] time = 0.69, size = 107, normalized size = 0.67

$$
\frac{1}{4}\left(a^4\left(-\frac{\sqrt{a^2x^2+1}\left(\left(\frac{1}{a^2x^2}-2\right)\sinh^{-1}(ax)^2+1\right)}{ax} - \sinh^{-1}(ax)\left(\frac{1}{a^2x^2}+2\sinh^{-1}(ax)+4\log\left(1-e^{-2\sinh^{-1}(ax)}\right)\right) +
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/x^5,x]
```

```
[Out] (-(ArcSinh[a*x]^3/x^4) + a^4*(-((Sqrt[1 + a^2*x^2]*(1 + (-2 + 1/(a^2*x^2))*
ArcSinh[a*x]^2))/(a*x)) - ArcSinh[a*x]*(1/(a^2*x^2) + 2*ArcSinh[a*x] + 4*Lo
g[1 - E^(-2*ArcSinh[a*x])]) + 2*PolyLog[2, E^(-2*ArcSinh[a*x])]))/4
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.47, size = 210, normalized size = 1.32

$$\frac{a^4 \operatorname{arcsinh}(ax)^2}{2}+\frac{a^3 \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1}}{2x}+\frac{a^4}{4}-\frac{a^3\sqrt{a^2x^2+1}}{4x}-\frac{a \operatorname{arcsinh}(ax)^2 \sqrt{a^2x^2+1}}{4x^3}-\frac{a^2 \operatorname{arcsinh}(ax}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^5,x)
```

```
[Out] 1/2*a^4*arcsinh(a*x)^2+1/2*a^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x+1/4*a^4-1
/4*a^3*(a^2*x^2+1)^(1/2)/x-1/4*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x^3-1/4*a
^2*arcsinh(a*x)/x^2-1/4*arcsinh(a*x)^3/x^4-a^4*arcsinh(a*x)*ln(1-a*x-(a^2*x
^2+1)^(1/2))-a^4*polylog(2,a*x+(a^2*x^2+1)^(1/2))-a^4*arcsinh(a*x)*ln(a*x+(
a^2*x^2+1)^(1/2)+1)-a^4*polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{4\,x^4}+\int \frac{3\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\left(a^3x^7+ax^5+\left(a^2x^6+x^4\right)\sqrt{a^2x^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*log(a*x + sqrt(a^2*x^2 + 1))^3/x^4 + integrate(3/4*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^7 + a*x^5 + (a^2
*x^6 + x^4)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^3/x^5,x)
```

```
[Out] int(asinh(a*x)^3/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**5,x)
```

```
[Out] Integral(asinh(a*x)**3/x**5, x)
```

3.32 $\int x^5 \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=276

$$\frac{5\sinh^{-1}(ax)^4}{96a^6}+\frac{245\sinh^{-1}(ax)^2}{1152a^6}+\frac{245x^2}{1152a^4}+\frac{5x^2\sinh^{-1}(ax)^2}{16a^4}-\frac{65x^4}{3456a^2}-\frac{5x^4\sinh^{-1}(ax)^2}{48a^2}-\frac{x^5\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{9a}$$

```
[Out] 245/1152*x^2/a^4-65/3456*x^4/a^2+1/324*x^6+245/1152*arcsinh(a*x)^2/a^6+5/16
*x^2*arcsinh(a*x)^2/a^4-5/48*x^4*arcsinh(a*x)^2/a^2+1/18*x^6*arcsinh(a*x)^2
+5/96*arcsinh(a*x)^4/a^6+1/6*x^6*arcsinh(a*x)^4-245/576*x*arcsinh(a*x)*(a^2
*x^2+1)^(1/2)/a^5+65/864*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-1/54*x^5*ar
csinh(a*x)*(a^2*x^2+1)^(1/2)/a-5/24*x*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^5+
5/36*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^3-1/9*x^5*arcsinh(a*x)^3*(a^2*x
^2+1)^(1/2)/a
```

Rubi [A] time = 0.86, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5661, 5758, 5675, 30}

$$-\frac{65x^4}{3456a^2}+\frac{245x^2}{1152a^4}-\frac{x^5\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{9a}-\frac{x^5\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{54a}-\frac{5x^4\sinh^{-1}(ax)^2}{48a^2}+\frac{5x^3\sqrt{a^2x^2+1}\,\sinh^{-1}}{36a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcSinh[a*x]^4,x]
```

```
[Out] (245*x^2)/(1152*a^4) - (65*x^4)/(3456*a^2) + x^6/324 - (245*x*Sqrt[1 + a^2*
x^2]*ArcSinh[a*x])/(576*a^5) + (65*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(864
*a^3) - (x^5*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(54*a) + (245*ArcSinh[a*x]^2)/
(1152*a^6) + (5*x^2*ArcSinh[a*x]^2)/(16*a^4) - (5*x^4*ArcSinh[a*x]^2)/(48*a
^2) + (x^6*ArcSinh[a*x]^2)/18 - (5*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(24*
a^5) + (5*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(36*a^3) - (x^5*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^3)/(9*a) + (5*ArcSinh[a*x]^4)/(96*a^6) + (x^6*ArcSinh[a
*x]^4)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
&& GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^5 \sinh^{-1}(ax)^4\,dx = \frac{1}{6}x^6 \sinh^{-1}(ax)^4 - \frac{1}{3}(2a)\int \frac{x^6 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{x^5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{9a} + \frac{1}{6}x^6 \sinh^{-1}(ax)^4 + \frac{1}{3}\int x^5 \sinh^{-1}(ax)^2\,dx + \frac{5\int \frac{x^4 \sinh^{-1}(}{\sqrt{1+a^2}}}{9a}$$

$$= \frac{1}{18}x^6 \sinh^{-1}(ax)^2 + \frac{5x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{36a^3} - \frac{x^5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{9a} + \frac{1}{6}x^6 \sinh$$

$$= -\frac{x^5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{54a} - \frac{5x^4 \sinh^{-1}(ax)^2}{48a^2} + \frac{1}{18}x^6 \sinh^{-1}(ax)^2 - \frac{5x\sqrt{1+a^2x^2}\,\sinh}{24a^5}$$

$$= \frac{x^6}{324} + \frac{65x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{864a^3} - \frac{x^5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{54a} + \frac{5x^2 \sinh^{-1}(ax)^2}{16a^4} - 5x$$

$$= -\frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{576a^5} + \frac{65x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{864a^3} - \frac{x^5\sqrt{1+}}{}$$

$$= \frac{245x^2}{1152a^4} - \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{576a^5} + \frac{65x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{864a^3} -$$

Mathematica [A] time = 0.11, size = 165, normalized size = 0.60

$$108\left(16a^6x^6+5\right)\sinh^{-1}(ax)^4 + a^2x^2\left(32a^4x^4-195a^2x^2+2205\right) - 144ax\sqrt{a^2x^2+1}\,\left(8a^4x^4-10a^2x^2+15\right)\sin$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcSinh[a*x]^4,x]
```

```
[Out] (a^2*x^2*(2205 - 195*a^2*x^2 + 32*a^4*x^4) - 6*a*x*Sqrt[1 + a^2*x^2]*(735 -
 130*a^2*x^2 + 32*a^4*x^4)*ArcSinh[a*x] + 9*(245 + 360*a^2*x^2 - 120*a^4*x^
4 + 64*a^6*x^6)*ArcSinh[a*x]^2 - 144*a*x*Sqrt[1 + a^2*x^2]*(15 - 10*a^2*x^2
 + 8*a^4*x^4)*ArcSinh[a*x]^3 + 108*(5 + 16*a^6*x^6)*ArcSinh[a*x]^4)/(10368*
a^6)
```

fricas [A] time = 0.41, size = 208, normalized size = 0.75

$$32\,a^6x^6 - 195\,a^4x^4 + 108\left(16\,a^6x^6+5\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^4 - 144\left(8\,a^5x^5-10\,a^3x^3+15\,ax\right)\sqrt{a^2x^2+1}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/10368*(32*a^6*x^6 - 195*a^4*x^4 + 108*(16*a^6*x^6 + 5)*log(a*x + sqrt(a^2
*x^2 + 1))^4 - 144*(8*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*sqrt(a^2*x^2 + 1)*log(
a*x + sqrt(a^2*x^2 + 1))^3 + 2205*a^2*x^2 + 9*(64*a^6*x^6 - 120*a^4*x^4 + 3
60*a^2*x^2 + 245)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(32*a^5*x^5 - 130*a^3*
x^3 + 735*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^6
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.48, size = 242, normalized size = 0.88

$$\frac{\frac{a^6x^6 \operatorname{arcsinh}(ax)^4}{6} - \frac{a^5x^5 \operatorname{arcsinh}(ax)^3 \sqrt{a^2x^2+1}}{9} + \frac{5a^3x^3 \operatorname{arcsinh}(ax)^3 \sqrt{a^2x^2+1}}{36} - \frac{5ax \operatorname{arcsinh}(ax)^3 \sqrt{a^2x^2+1}}{24} + \frac{5 \operatorname{arcsinh}(ax)^4}{96} + \frac{\operatorname{arcsinh}(ax)^2}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^6*(1/6*a^6*x^6*arcsinh(a*x)^4-1/9*a^5*x^5*arcsinh(a*x)^3*(a^2*x^2+1)^(1
/2)+5/36*a^3*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-5/24*a*x*arcsinh(a*x)^3*(
a^2*x^2+1)^(1/2)+5/96*arcsinh(a*x)^4+1/18*arcsinh(a*x)^2*a^6*x^6-1/54*arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)*a^5*x^5+65/864*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(
1/2)-245/576*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-115/1152*arcsinh(a*x)^2+1/3
24*a^6*x^6-65/3456*a^4*x^4+245/1152*a^2*x^2+245/1152-5/48*a^4*x^4*arcsinh(a
*x)^2+5/16*(a^2*x^2+1)*arcsinh(a*x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,x^6 \log\left(ax + \sqrt{a^2x^2+1}\right)^4 - \int \frac{2\left(a^3x^8 + \sqrt{a^2x^2+1}\,a^2x^7 + ax^6\right)\log\left(ax + \sqrt{a^2x^2+1}\right)^3}{3\left(a^3x^3 + ax + \left(a^2x^2+1\right)^{\frac{3}{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate(2/3*(a^3*x^8 + sqrt(a^2*
x^2 + 1)*a^2*x^7 + a*x^6)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \operatorname{asinh}(a\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*asinh(a*x)^4,x)
```

```
[Out] int(x^5*asinh(a*x)^4, x)
```

sympy [A] time = 15.20, size = 269, normalized size = 0.97

$$\begin{cases} \frac{x^6 \operatorname{asinh}^4(ax)}{6} + \frac{x^6 \operatorname{asinh}^2(ax)}{18} + \frac{x^6}{324} - \frac{x^5\sqrt{a^2x^2+1}\,\operatorname{asinh}^3(ax)}{9a} - \frac{x^5\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{54a} - \frac{5x^4 \operatorname{asinh}^2(ax)}{48a^2} - \frac{65x^4}{3456a^2} + \frac{5x^3\sqrt{a^2x^2+1}\,\operatorname{asin}}{36a^3} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**6*asinh(a*x)**4/6 + x**6*asinh(a*x)**2/18 + x**6/324 - x**5*s
qrt(a**2*x**2 + 1)*asinh(a*x)**3/(9*a) - x**5*sqrt(a**2*x**2 + 1)*asinh(a*x
```

```
)/(54*a) - 5*x**4*asinh(a*x)**2/(48*a**2) - 65*x**4/(3456*a**2) + 5*x**3*sq
rt(a**2*x**2 + 1)*asinh(a*x)**3/(36*a**3) + 65*x**3*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(864*a**3) + 5*x**2*asinh(a*x)**2/(16*a**4) + 245*x**2/(1152*a**4)
- 5*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(24*a**5) - 245*x*sqrt(a**2*x**2 +
1)*asinh(a*x)/(576*a**5) + 5*asinh(a*x)**4/(96*a**6) + 245*asinh(a*x)**2/(1
152*a**6), Ne(a, 0)), (0, True))
```

3.33 $\int x^4 \sinh^{-1}(ax)^4\, dx$

Optimal. Leaf size=244

$$\frac{16576x}{5625a^4}+\frac{32x\sinh^{-1}(ax)^2}{25a^4}-\frac{1088x^3}{16875a^2}-\frac{16x^3\sinh^{-1}(ax)^2}{75a^2}-\frac{4x^4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{25a}-\frac{24x^4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{625a}$$

```
[Out] 16576/5625*x/a^4-1088/16875*x^3/a^2+24/3125*x^5+32/25*x*arcsinh(a*x)^2/a^4-
16/75*x^3*arcsinh(a*x)^2/a^2+12/125*x^5*arcsinh(a*x)^2+1/5*x^5*arcsinh(a*x)
^4-16576/5625*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^5+1088/5625*x^2*arcsinh(a*x)
*(a^2*x^2+1)^(1/2)/a^3-24/625*x^4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a-32/75*ar
csinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^5+16/75*x^2*arcsinh(a*x)^3*(a^2*x^2+1)^(1/
2)/a^3-4/25*x^4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.66, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5661, 5758, 5717, 5653, 8, 30}

$$-\frac{1088x^3}{16875a^2}-\frac{4x^4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{25a}-\frac{24x^4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{625a}-\frac{16x^3\sinh^{-1}(ax)^2}{75a^2}+\frac{16x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(a}{75a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^4,x]
```

```
[Out] (16576*x)/(5625*a^4) - (1088*x^3)/(16875*a^2) + (24*x^5)/3125 - (16576*Sqrt
[1 + a^2*x^2]*ArcSinh[a*x])/(5625*a^5) + (1088*x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x])/(5625*a^3) - (24*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(625*a) + (32*
x*ArcSinh[a*x]^2)/(25*a^4) - (16*x^3*ArcSinh[a*x]^2)/(75*a^2) + (12*x^5*Arc
Sinh[a*x]^2)/125 - (32*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(75*a^5) + (16*x^2
*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[1 + a^2*x^2]*ArcS
inh[a*x]^3)/(25*a) + (x^5*ArcSinh[a*x]^4)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
```

```
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^4\,dx = \frac{1}{5}x^5\sinh^{-1}(ax)^4 - \frac{1}{5}(4a)\int \frac{x^5\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{4x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{25a} + \frac{1}{5}x^5\sinh^{-1}(ax)^4 + \frac{12}{25}\int x^4\sinh^{-1}(ax)^2\,dx + \frac{16\int \frac{x^3\sin}{\sqrt{1}}}{2}$$

$$= \frac{12}{125}x^5\sinh^{-1}(ax)^2 + \frac{16x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{75a^3} - \frac{4x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{25a} + \frac{1}{5}x^5$$

$$= -\frac{24x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{625a} - \frac{16x^3\sinh^{-1}(ax)^2}{75a^2} + \frac{12}{125}x^5\sinh^{-1}(ax)^2 - \frac{32\sqrt{1+a^2x^2}}{75}$$

$$= \frac{24x^5}{3125} + \frac{1088x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{5625a^3} - \frac{24x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{625a} + \frac{32x\sinh^{-1}(ax)^2}{25a^4}$$

$$= -\frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{5625a^5} + \frac{1088x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{5625a^3} - 2$$

$$= \frac{16576x}{5625a^4} - \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{5625a^5} + \frac{1088x^2\sqrt{1+a^2x^2}\,\sinh^{-1}}{5625a^3}$$

Mathematica [A] time = 0.09, size = 148, normalized size = 0.61

$$\frac{16875a^5x^5\sinh^{-1}(ax)^4 + 8ax\left(81a^4x^4 - 680a^2x^2 + 31080\right) + 900ax\left(9a^4x^4 - 20a^2x^2 + 120\right)\sinh^{-1}(ax)^2 - 450}{84375a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x]^4,x]
```

```
[Out] (8*a*x*(31080 - 680*a^2*x^2 + 81*a^4*x^4) - 120*Sqrt[1 + a^2*x^2]*(2072 - 1
36*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] + 900*a*x*(120 - 20*a^2*x^2 + 9*a^4*x
^4)*ArcSinh[a*x]^2 - 4500*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 + 3*a^4*x^4)*Arc
Sinh[a*x]^3 + 16875*a^5*x^5*ArcSinh[a*x]^4)/(84375*a^5)
```

fricas [A] time = 0.43, size = 189, normalized size = 0.77

$$16875\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 648\,a^5x^5 - 5440\,a^3x^3 - 4500\left(3\,a^4x^4 - 4\,a^2x^2 + 8\right)\sqrt{a^2x^2+1}\,\log\left(ax+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/84375*(16875*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^4 + 648*a^5*x^5 - 5440*
a^3*x^3 - 4500*(3*a^4*x^4 - 4*a^2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 + 1))^3 + 900*(9*a^5*x^5 - 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt(a^
2*x^2 + 1))^2 - 120*(27*a^4*x^4 - 136*a^2*x^2 + 2072)*sqrt(a^2*x^2 + 1)*log
(a*x + sqrt(a^2*x^2 + 1)) + 248640*a*x)/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.48, size = 210, normalized size = 0.86

$$\frac{\frac{a^5x^5\operatorname{arcsinh}(ax)^4}{5}-\frac{32\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{75}-\frac{4a^4x^4\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{25}+\frac{16a^2x^2\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{75}+\frac{32ax\operatorname{arcsinh}(ax)^2}{25}-16576\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)^4-32/75*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-4/
25*a^4*x^4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+16/75*a^2*x^2*arcsinh(a*x)^3*(a
^2*x^2+1)^(1/2)+32/25*a*x*arcsinh(a*x)^2-16576/5625*(a^2*x^2+1)^(1/2)*arcsi
nh(a*x)+16576/5625*a*x+12/125*a^5*x^5*arcsinh(a*x)^2-24/625*a^4*x^4*arcsinh
(a*x)*(a^2*x^2+1)^(1/2)+1088/5625*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+24
/3125*a^5*x^5-1088/16875*a^3*x^3-16/75*a^3*x^3*arcsinh(a*x)^2)
```

maxima [A] time = 0.35, size = 201, normalized size = 0.82

$$\frac{1}{5}x^5\operatorname{arsinh}(ax)^4-\frac{4}{75}\left(\frac{3\sqrt{a^2x^2+1}\,x^4}{a^2}-\frac{4\sqrt{a^2x^2+1}\,x^2}{a^4}+\frac{8\sqrt{a^2x^2+1}}{a^6}\right)a\operatorname{arsinh}(ax)^3-\frac{4}{84375}\left(2a\left(\frac{15\left(27\sqrt{a^2x}\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^4 - 4/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x)^3 - 4/84375*(2*a*(1
5*(27*sqrt(a^2*x^2 + 1)*a^2*x^4 - 136*sqrt(a^2*x^2 + 1)*x^2 + 2072*sqrt(a^2
*x^2 + 1)/a^2)*arcsinh(a*x)/a^5 - (81*a^4*x^5 - 680*a^2*x^3 + 31080*x)/a^6)
 - 225*(9*a^4*x^5 - 20*a^2*x^3 + 120*x)*arcsinh(a*x)^2/a^5)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\operatorname{asinh}(a\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asinh(a*x)^4,x)
```

```
[Out] int(x^4*asinh(a*x)^4, x)
```

sympy [A] time = 8.86, size = 241, normalized size = 0.99

$$\begin{cases} \frac{x^5 \operatorname{asinh}^4(ax)}{5} + \frac{12x^5 \operatorname{asinh}^2(ax)}{125} + \frac{24x^5}{3125} - \frac{4x^4\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{25a} - \frac{24x^4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{625a} - \frac{16x^3\operatorname{asinh}^2(ax)}{75a^2} - \frac{1088x^3}{16875a^2} + \frac{16x^2 \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**4/5 + 12*x**5*asinh(a*x)**2/125 + 24*x**5/3125
- 4*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(25*a) - 24*x**4*sqrt(a**2*x**2
+ 1)*asinh(a*x)/(625*a) - 16*x**3*asinh(a*x)**2/(75*a**2) - 1088*x**3/(1687
5*a**2) + 16*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(75*a**3) + 1088*x**2*s
qrt(a**2*x**2 + 1)*asinh(a*x)/(5625*a**3) + 32*x*asinh(a*x)**2/(25*a**4) +
16576*x/(5625*a**4) - 32*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(75*a**5) - 1657
6*sqrt(a**2*x**2 + 1)*asinh(a*x)/(5625*a**5), Ne(a, 0)), (0, True))
```

3.34 $\int x^3 \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=194

$$-\frac{3\sinh^{-1}(ax)^4}{32a^4}-\frac{45\sinh^{-1}(ax)^2}{128a^4}-\frac{45x^2}{128a^2}-\frac{9x^2\sinh^{-1}(ax)^2}{16a^2}-\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{4a}-\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{32a}$$

```
[Out] -45/128*x^2/a^2+3/128*x^4-45/128*arcsinh(a*x)^2/a^4-9/16*x^2*arcsinh(a*x)^2
/a^2+3/16*x^4*arcsinh(a*x)^2-3/32*arcsinh(a*x)^4/a^4+1/4*x^4*arcsinh(a*x)^4
+45/64*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-3/32*x^3*arcsinh(a*x)*(a^2*x^2+
1)^(1/2)/a+3/8*x*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^3-1/4*x^3*arcsinh(a*x)^
3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.50, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5661, 5758, 5675, 30}

$$-\frac{45x^2}{128a^2}-\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{4a}-\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{32a}-\frac{9x^2\sinh^{-1}(ax)^2}{16a^2}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{8a^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^4,x]
```

```
[Out] (-45*x^2)/(128*a^2) + (3*x^4)/128 + (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(
64*a^3) - (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a) - (45*ArcSinh[a*x]^
2)/(128*a^4) - (9*x^2*ArcSinh[a*x]^2)/(16*a^2) + (3*x^4*ArcSinh[a*x]^2)/16
+ (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^3) - (x^3*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^3)/(4*a) - (3*ArcSinh[a*x]^4)/(32*a^4) + (x^4*ArcSinh[a*x]^4)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^4\,dx = \frac{1}{4}x^4\sinh^{-1}(ax)^4 - a\int \frac{x^4\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{4a} + \frac{1}{4}x^4\sinh^{-1}(ax)^4 + \frac{3}{4}\int x^3\sinh^{-1}(ax)^2\,dx + \frac{3\int \frac{x^2\sinh^{-1}(}{\sqrt{1+a^2}}}{4a}$$
$$= \frac{3}{16}x^4\sinh^{-1}(ax)^2 + \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{8a^3} - \frac{x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{4a} + \frac{1}{4}x^4\sinh$$
$$= -\frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a} - \frac{9x^2\sinh^{-1}(ax)^2}{16a^2} + \frac{3}{16}x^4\sinh^{-1}(ax)^2 + \frac{3x\sqrt{1+a^2x^2}\,\sin}{8a^3}$$
$$= \frac{3x^4}{128} + \frac{45x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a} - \frac{9x^2\sinh^{-1}(ax)^2}{16a^2} + \frac{3}{16}$$
$$= -\frac{45x^2}{128a^2} + \frac{3x^4}{128} + \frac{45x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{32a} - \frac{45\sinh^{-1}(}{128a^4}$$

Mathematica [A] time = 0.07, size = 133, normalized size = 0.69

$$\frac{4\left(8a^4x^4-3\right)\sinh^{-1}(ax)^4 + 3a^2x^2\left(a^2x^2-15\right) - 16ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)^3 - 6ax\sqrt{a^2x^2+1}\left(2a^2\right.}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x]^4,x]
```

```
[Out] (3*a^2*x^2*(-15 + a^2*x^2) - 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*ArcS
inh[a*x] + 3*(-15 - 24*a^2*x^2 + 8*a^4*x^4)*ArcSinh[a*x]^2 - 16*a*x*Sqrt[1
+ a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 4*(-3 + 8*a^4*x^4)*ArcSinh[a*x
]^4)/(128*a^4)
```

fricas [A] time = 0.44, size = 176, normalized size = 0.91

$$\frac{3\,a^4x^4 + 4\left(8\,a^4x^4-3\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^4 - 16\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - 45\,a^2x^2}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(3*a^4*x^4 + 4*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^4 - 16*(2
*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 45*a^2
*x^2 + 3*(8*a^4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(
2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.38, size = 172, normalized size = 0.89

$$\frac{\frac{a^4x^4\operatorname{arcsinh}(ax)^4}{4}-\frac{a^3x^3\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{4}+\frac{3ax\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{8}-\frac{3\operatorname{arcsinh}(ax)^4}{32}+\frac{3a^4x^4\operatorname{arcsinh}(ax)^2}{16}-\frac{3a^3x^3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{32}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)^4-1/4*a^3*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1
/2)+3/8*a*x*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-3/32*arcsinh(a*x)^4+3/16*a^4*x
^4*arcsinh(a*x)^2-3/32*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+45/64*arcsinh
(a*x)*(a^2*x^2+1)^(1/2)*a*x+27/128*arcsinh(a*x)^2+3/128*a^4*x^4-45/128*a^2*
x^2-45/128-9/16*(a^2*x^2+1)*arcsinh(a*x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^4\log\left(ax+\sqrt{a^2x^2+1}\right)^4-\int\frac{\left(a^3x^6+\sqrt{a^2x^2+1}\,a^2x^5+ax^4\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^3x^3+ax+\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate((a^3*x^6 + sqrt(a^2*x^2
+ 1)*a^2*x^5 + a*x^4)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (a^2*
x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{asinh}\left(a\,x\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x)^4,x)
```

```
[Out] int(x^3*asinh(a*x)^4, x)
```

sympy [A] time = 5.56, size = 190, normalized size = 0.98

$$\begin{cases}\frac{x^4\operatorname{asinh}^4(ax)}{4}+\frac{3x^4\operatorname{asinh}^2(ax)}{16}+\frac{3x^4}{128}-\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a}-\frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a}-\frac{9x^2\operatorname{asinh}^2(ax)}{16a^2}-\frac{45x^2}{128a^2}+\frac{3x\sqrt{a^2x^2+1}\operatorname{asin}}{8a^3} \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**4/4 + 3*x**4*asinh(a*x)**2/16 + 3*x**4/128 - x*
*3*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(4*a) - 3*x**3*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(32*a) - 9*x**2*asinh(a*x)**2/(16*a**2) - 45*x**2/(128*a**2) + 3*x*
sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(8*a**3) + 45*x*sqrt(a**2*x**2 + 1)*asinh
(a*x)/(64*a**3) - 3*asinh(a*x)**4/(32*a**4) - 45*asinh(a*x)**2/(128*a**4),
Ne(a, 0)), (0, True))
```

3.35 $\int x^2 \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=162

$$-\frac{4x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{9a}-\frac{8x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{27a}-\frac{160x}{27a^2}-\frac{8x\sinh^{-1}(ax)^2}{3a^2}+\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{9a^3}+\frac{16\cdots}{}$$

```
[Out] -160/27*x/a^2+8/81*x^3-8/3*x*arcsinh(a*x)^2/a^2+4/9*x^3*arcsinh(a*x)^2+1/3*
x^3*arcsinh(a*x)^4+160/27*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-8/27*x^2*arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)/a+8/9*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^3-4/9*x^
2*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.36, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5661, 5758, 5717, 5653, 8, 30}

$$-\frac{4x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{9a}+\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{9a^3}-\frac{8x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{27a}+\frac{160\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{27a^3}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^4,x]
```

```
[Out] (-160*x)/(27*a^2) + (8*x^3)/81 + (160*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a
^3) - (8*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a) - (8*x*ArcSinh[a*x]^2)/
(3*a^2) + (4*x^3*ArcSinh[a*x]^2)/9 + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(
9*a^3) - (4*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(9*a) + (x^3*ArcSinh[a*x]
^4)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\sinh^{-1}(ax)^4\,dx = \frac{1}{3}x^3\sinh^{-1}(ax)^4 - \frac{1}{3}(4a)\int\frac{x^3\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{4x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{9a} + \frac{1}{3}x^3\sinh^{-1}(ax)^4 + \frac{4}{3}\int x^2\sinh^{-1}(ax)^2\,dx + \frac{8\int\frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}}{9a}$$
$$= \frac{4}{9}x^3\sinh^{-1}(ax)^2 + \frac{8\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{9a^3} - \frac{4x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{9a} + \frac{1}{3}x^3\sinh^{-1}(a$$
$$= -\frac{8x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{27a} - \frac{8x\sinh^{-1}(ax)^2}{3a^2} + \frac{4}{9}x^3\sinh^{-1}(ax)^2 + \frac{8\sqrt{1+a^2x^2}\,\sinh^{-1}(ax}{9a^3}$$
$$= \frac{8x^3}{81} + \frac{160\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{27a} - \frac{8x\sinh^{-1}(ax)^2}{3a^2} + \frac{4}{9}x^3\sin$$
$$= -\frac{160x}{27a^2} + \frac{8x^3}{81} + \frac{160\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{27a^3} - \frac{8x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{27a} - \frac{8x\sinh^{-1}(ax)^2}{3a^2}$$

Mathematica [A] time = 0.07, size = 112, normalized size = 0.69

$$\frac{27a^3x^3\sinh^{-1}(ax)^4 + 8ax\left(a^2x^2-60\right) - 36\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3 + 36ax\left(a^2x^2-6\right)\sinh^{-1}(ax)^2 - 24}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x]^4,x]
```

```
[Out] (8*a*x*(-60 + a^2*x^2) - 24*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
+ 36*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 - 36*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2
]*ArcSinh[a*x]^3 + 27*a^3*x^3*ArcSinh[a*x]^4)/(81*a^3)
```

fricas [A] time = 0.42, size = 154, normalized size = 0.95

$$\frac{27\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^4 + 8\,a^3x^3 - 36\,\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 36\left(a^3x^3-6\,ax\right)\log}{81\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/81*(27*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^4 + 8*a^3*x^3 - 36*sqrt(a^2*x
^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 36*(a^3*x^3 - 6*a*x)
*log(a*x + sqrt(a^2*x^2 + 1))^2 - 24*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20)*log(a
*x + sqrt(a^2*x^2 + 1)) - 480*a*x)/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.38, size = 140, normalized size = 0.86

$$\frac{\frac{a^3x^3\operatorname{arcsinh}(ax)^4}{3}+\frac{8\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{9}-\frac{4a^2x^2\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{9}-\frac{8ax\operatorname{arcsinh}(ax)^2}{3}+\frac{160\sqrt{a^2x^2+1}\,\operatorname{arcsinh}(ax)}{27}-\frac{160ax}{27}+}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)^4+8/9*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-4/9*
a^2*x^2*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-8/3*a*x*arcsinh(a*x)^2+160/27*(a^2
*x^2+1)^(1/2)*arcsinh(a*x)-160/27*a*x+4/9*a^3*x^3*arcsinh(a*x)^2-8/27*arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+8/81*a^3*x^3)
```

maxima [A] time = 0.33, size = 143, normalized size = 0.88

$$\frac{1}{3}x^3\operatorname{arsinh}(ax)^4-\frac{4}{9}a\left(\frac{\sqrt{a^2x^2+1}\,x^2}{a^2}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^3-\frac{4}{81}\left(2a\left(\frac{3\left(\sqrt{a^2x^2+1}\,x^2-\frac{20\sqrt{a^2x^2+1}}{a^2}\right)\operatorname{arsi}}{a^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^4 - 4/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x)^3 - 4/81*(2*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^
2*x^2 + 1)/a^2)*arcsinh(a*x)/a^3 - (a^2*x^3 - 60*x)/a^4) - 9*(a^2*x^3 - 6*x
)*arcsinh(a*x)^2/a^3)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\operatorname{asinh}(a\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x)^4,x)
```

```
[Out] int(x^2*asinh(a*x)^4, x)
```

sympy [A] time = 3.08, size = 158, normalized size = 0.98

$$\begin{cases}\frac{x^3\operatorname{asinh}^4(ax)}{3}+\frac{4x^3\operatorname{asinh}^2(ax)}{9}+\frac{8x^3}{81}-\frac{4x^2\sqrt{a^2x^2+1}\,\operatorname{asinh}^3(ax)}{9a}-\frac{8x^2\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}{27a}-\frac{8x\operatorname{asinh}^2(ax)}{3a^2}-\frac{160x}{27a^2}+\frac{8\sqrt{a^2x^2+1}\,\operatorname{as}}{9a^3}\\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**4/3 + 4*x**3*asinh(a*x)**2/9 + 8*x**3/81 - 4*x*
*2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(9*a) - 8*x**2*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(27*a) - 8*x*asinh(a*x)**2/(3*a**2) - 160*x/(27*a**2) + 8*sqrt(a**2
*x**2 + 1)*asinh(a*x)**3/(9*a**3) + 160*sqrt(a**2*x**2 + 1)*asinh(a*x)/(27*
a**3), Ne(a, 0)), (0, True))
```

3.36 $\int x \sinh^{-1}(ax)^4 \, dx$

Optimal. Leaf size=110

$$-\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{a}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a}+\frac{\sinh^{-1}(ax)^4}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^4+\frac{3}{2}x^2\sinh^{-1}$$

```
[Out] 3/4*x^2+3/4*arcsinh(a*x)^2/a^2+3/2*x^2*arcsinh(a*x)^2+1/4*arcsinh(a*x)^4/a^
2+1/2*x^2*arcsinh(a*x)^4-3/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a-x*arcsinh(a
*x)^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.24, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5661, 5758, 5675, 30}

$$-\frac{x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{a}-\frac{3x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{2a}+\frac{\sinh^{-1}(ax)^4}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{4a^2}+\frac{1}{2}x^2\sinh^{-1}(ax)^4+\frac{3}{2}x^2\sinh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^4,x]
```

```
[Out] (3*x^2)/4 - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a) + (3*ArcSinh[a*x]^2)
/(4*a^2) + (3*x^2*ArcSinh[a*x]^2)/2 - (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/
a + ArcSinh[a*x]^4/(4*a^2) + (x^2*ArcSinh[a*x]^4)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x\sinh^{-1}(ax)^4\,dx = \frac{1}{2}x^2\sinh^{-1}(ax)^4 - (2a)\int \frac{x^2\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a} + \frac{1}{2}x^2\sinh^{-1}(ax)^4 + 3\int x\sinh^{-1}(ax)^2\,dx + \frac{\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{a}$$
$$= \frac{3}{2}x^2\sinh^{-1}(ax)^2 - \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a} + \frac{\sinh^{-1}(ax)^4}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^4 - (3a)\int$$
$$= -\frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a} + \frac{3}{2}x^2\sinh^{-1}(ax)^2 - \frac{x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a} + \frac{\sinh^{-1}(ax)^4}{4a^2}$$
$$= \frac{3x^2}{4} - \frac{3x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{2a} + \frac{3\sinh^{-1}(ax)^2}{4a^2} + \frac{3}{2}x^2\sinh^{-1}(ax)^2 - \frac{x\sqrt{1+a^2x^2}\,\sinh}{a}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 0.85

$$\frac{3a^2x^2 + \left(2a^2x^2+1\right)\sinh^{-1}(ax)^4 - 4ax\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3 + \left(6a^2x^2+3\right)\sinh^{-1}(ax)^2 - 6ax\sqrt{a^2x^2+1}\,\sinh}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x]^4,x]
```

```
[Out] (3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*ArcSinh
[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + (1 + 2*a^2*x^2)*ArcSinh[
a*x]^4)/(4*a^2)
```

fricas [A] time = 0.41, size = 138, normalized size = 1.25

$$-\frac{4\sqrt{a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - \left(2a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^4 - 3a^2x^2 + 6\sqrt{a^2x^2+1}\,ax\log\left(\right.}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - (2*a^2*x^2 +
 1)*log(a*x + sqrt(a^2*x^2 + 1))^4 - 3*a^2*x^2 + 6*sqrt(a^2*x^2 + 1)*a*x*lo
g(a*x + sqrt(a^2*x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
^2)/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.10, size = 105, normalized size = 0.95

$$\frac{\frac{\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^4}{2} - ax\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1} - \frac{\operatorname{arcsinh}(ax)^4}{4} + \frac{3\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)^2}{2} - \frac{3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{2} - \frac{3\,\mathrm{ar}}{}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^4,x)
```

```
[Out] 1/a^2*(1/2*(a^2*x^2+1)*arcsinh(a*x)^4-a*x*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-
1/4*arcsinh(a*x)^4+3/2*(a^2*x^2+1)*arcsinh(a*x)^2-3/2*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)*a*x-3/4*arcsinh(a*x)^2+3/4*a^2*x^2+3/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2\log\left(ax+\sqrt{a^2x^2+1}\right)^4-\int\frac{2\left(a^3x^4+\sqrt{a^2x^2+1}\,a^2x^3+ax^2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^3x^3+ax+\left(a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate(2*(a^3*x^4 + sqrt(a^2*x^
2 + 1)*a^2*x^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (a^
2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{asinh}\left(a\,x\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x)^4,x)
```

```
[Out] int(x*asinh(a*x)^4, x)
```

sympy [A] time = 1.78, size = 104, normalized size = 0.95

$$\begin{cases}\frac{x^2\operatorname{asinh}^4(ax)}{2}+\frac{3x^2\operatorname{asinh}^2(ax)}{2}+\frac{3x^2}{4}-\frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{a}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a}+\frac{\operatorname{asinh}^4(ax)}{4a^2}+\frac{3\operatorname{asinh}^2(ax)}{4a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**4/2 + 3*x**2*asinh(a*x)**2/2 + 3*x**2/4 - x*sqr
t(a**2*x**2 + 1)*asinh(a*x)**3/a - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a)
 + asinh(a*x)**4/(4*a**2) + 3*asinh(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

3.37 $\int \sinh^{-1}(ax)^4\,dx$

Optimal. Leaf size=67

$$-\frac{4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{a}-\frac{24\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^4+12x\sinh^{-1}(ax)^2+24x$$

```
[Out] 24*x+12*x*arcsinh(a*x)^2+x*arcsinh(a*x)^4-24*arcsinh(a*x)*(a^2*x^2+1)^(1/2)
/a-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5653, 5717, 8}

$$-\frac{4\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{a}-\frac{24\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^4+12x\sinh^{-1}(ax)^2+24x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4,x]
```

```
[Out] 24*x - (24*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + 12*x*ArcSinh[a*x]^2 - (4*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^3)/a + x*ArcSinh[a*x]^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}(ax)^4\,dx &= x\sinh^{-1}(ax)^4-(4a)\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a}+x\sinh^{-1}(ax)^4+12\int \sinh^{-1}(ax)^2\,dx\\
&= 12x\sinh^{-1}(ax)^2-\frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a}+x\sinh^{-1}(ax)^4-(24a)\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{24\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}+12x\sinh^{-1}(ax)^2-\frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a}+x\sinh^{-1}(ax)^4\\
&= 24x-\frac{24\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}{a}+12x\sinh^{-1}(ax)^2-\frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{a}+x\sinh^{-1}(
\end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.00

$$-\frac{4\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{a}-\frac{24\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a}+x\sinh^{-1}(ax)^4+12x\sinh^{-1}(ax)^2+24x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^4,x]
```

```
[Out] 24*x - (24*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + 12*x*ArcSinh[a*x]^2 - (4*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^3)/a + x*ArcSinh[a*x]^4
```

fricas [A] time = 0.41, size = 112, normalized size = 1.67

$$\frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^4+12\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-4\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3+24\,ax-24\,\sqrt{a^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^4 + 12*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2
 - 4*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 24*a*x - 24*sqrt(a^
2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a
```

giac [A] time = 0.24, size = 125, normalized size = 1.87

$$x\log\left(ax+\sqrt{a^2x^2+1}\right)^4-4\left(\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2}-\frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2+1}\,\mathrm{l}}{}\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^4 - 4*(sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*
x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2
*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2))/a)*a
```

maple [A] time = 0.10, size = 65, normalized size = 0.97

$$\frac{ax\operatorname{arcsinh}(ax)^4-4\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}+12ax\operatorname{arcsinh}(ax)^2-24\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)+24ax}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4,x)
```

```
[Out] 1/a*(a*x*arcsinh(a*x)^4-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+12*a*x*arcsinh(a
*x)^2-24*(a^2*x^2+1)^(1/2)*arcsinh(a*x)+24*a*x)
```

maxima [A] time = 0.31, size = 73, normalized size = 1.09

$$x\operatorname{arsinh}(ax)^4-\frac{4\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^3}{a}+12\left(\frac{x\operatorname{arsinh}(ax)^2}{a}+\frac{2\left(x-\frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a}\right)}{a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^4 - 4*sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a + 12*(x*arcsinh(a*x
)^2/a + 2*(x - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a)/a)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^4,x)
```

```
[Out] int(asinh(a*x)^4, x)
```

sympy [A] time = 0.83, size = 65, normalized size = 0.97

$$\begin{cases} x\operatorname{asinh}^{4}(ax) + 12x\operatorname{asinh}^{2}(ax) + 24x - \frac{4\sqrt{a^{2}x^{2}+1}\operatorname{asinh}^{3}(ax)}{a} - \frac{24\sqrt{a^{2}x^{2}+1}\operatorname{asinh}(ax)}{a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4,x)
```

```
[Out] Piecewise((x*asinh(a*x)**4 + 12*x*asinh(a*x)**2 + 24*x - 4*sqrt(a**2*x**2 +
 1)*asinh(a*x)**3/a - 24*sqrt(a**2*x**2 + 1)*asinh(a*x)/a, Ne(a, 0)), (0, T
rue))
```

3.38 $\int \frac{\sinh^{-1}(ax)^4}{x}\,dx$

Optimal. Leaf size=97

$$2\sinh^{-1}(ax)^3\mathrm{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-3\sinh^{-1}(ax)^2\mathrm{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)\mathrm{Li}_4\left(e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}\mathrm{Li}_5\left(e^{2\sinh^{-1}(ax)}\right)-$$

```
[Out] -1/5*arcsinh(a*x)^5+arcsinh(a*x)^4*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+2*arcsin
h(a*x)^3*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3*arcsinh(a*x)^2*polylog(3,(a
*x+(a^2*x^2+1)^(1/2))^2)+3*arcsinh(a*x)*polylog(4,(a*x+(a^2*x^2+1)^(1/2))^2
)-3/2*polylog(5,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5659, 3716, 2190, 2531, 6609, 2282, 6589}

$$2\sinh^{-1}(ax)^3\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-3\sinh^{-1}(ax)^2\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)\mathrm{PolyLog}\left(4,e^{2\sinh^{-1}(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4/x,x]
```

```
[Out] -ArcSinh[a*x]^5/5 + ArcSinh[a*x]^4*Log[1 - E^(2*ArcSinh[a*x])] + 2*ArcSinh[
a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*ArcSinh[a*x]^2*PolyLog[3, E^(2*Ar
cSinh[a*x])] + 3*ArcSinh[a*x]*PolyLog[4, E^(2*ArcSinh[a*x])] - (3*PolyLog[5
, E^(2*ArcSinh[a*x])])/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^4}{x}\,dx &= \text{Subst}\left(\int x^4\coth(x)\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 - 2\,\text{Subst}\left(\int \frac{e^{2x}x^4}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) - 4\,\text{Subst}\left(\int x^3\log\left(1-e^{2x}\right)\,dx, x, \text{s}\right.\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 6\,\text{S}\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\,\text{s}\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\,\text{s}\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\,\text{s}\\
&= -\frac{1}{5}\sinh^{-1}(ax)^5 + \sinh^{-1}(ax)^4\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)^3\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\,\text{s}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 97, normalized size = 1.00

$$2\sinh^{-1}(ax)^3\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\text{Li}_3\left(e^{2\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)\text{Li}_4\left(e^{2\sinh^{-1}(ax)}\right) - \frac{3}{2}\text{Li}_5\left(e^{2\sinh^{-1}(ax)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^4/x,x]
```

```
[Out] -1/5*ArcSinh[a*x]^5 + ArcSinh[a*x]^4*Log[1 - E^(2*ArcSinh[a*x])] + 2*ArcSin
h[a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*ArcSinh[a*x]^2*PolyLog[3, E^(2*
ArcSinh[a*x])] + 3*ArcSinh[a*x]*PolyLog[4, E^(2*ArcSinh[a*x])] - (3*PolyLog
[5, E^(2*ArcSinh[a*x])])/2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^4}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^4/x, x)
```

maple [A] time = 0.16, size = 257, normalized size = 2.65

$$-\frac{\operatorname{arcsinh}(ax)^5}{5}+\operatorname{arcsinh}(ax)^4\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+4\operatorname{arcsinh}(ax)^3\operatorname{polylog}\left(2,ax+\sqrt{a^2x^2+1}\right)-12\operatorname{arcsin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4/x,x)
```

```
[Out] -1/5*arcsinh(a*x)^5+arcsinh(a*x)^4*ln(1-a*x-(a^2*x^2+1)^(1/2))+4*arcsinh(a*
x)^3*polylog(2,a*x+(a^2*x^2+1)^(1/2))-12*arcsinh(a*x)^2*polylog(3,a*x+(a^2*
x^2+1)^(1/2))+24*arcsinh(a*x)*polylog(4,a*x+(a^2*x^2+1)^(1/2))-24*polylog(5
,a*x+(a^2*x^2+1)^(1/2))+arcsinh(a*x)^4*ln(a*x+(a^2*x^2+1)^(1/2)+1)+4*arcsin
h(a*x)^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-12*arcsinh(a*x)^2*polylog(3,-a*x
-(a^2*x^2+1)^(1/2))+24*arcsinh(a*x)*polylog(4,-a*x-(a^2*x^2+1)^(1/2))-24*po
lylog(5,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^4/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^4/x,x)
```

```
[Out] int(asinh(a*x)^4/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^4(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4/x,x)
```

```
[Out] Integral(asinh(a*x)**4/x, x)
```

3.39 $\int \frac{\sinh^{-1}(ax)^4}{x^2}\,dx$

Optimal. Leaf size=120

$$-12a\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+12a\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)+24a\sinh^{-1}(ax)\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)-24a\sinh$$

```
[Out] -arcsinh(a*x)^4/x-8*a*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-12*a*ar
csinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+12*a*arcsinh(a*x)^2*polylog(
2,a*x+(a^2*x^2+1)^(1/2))+24*a*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2)
)-24*a*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))-24*a*polylog(4,-a*x-(a
^2*x^2+1)^(1/2))+24*a*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.19, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5661, 5760, 4182, 2531, 6609, 2282, 6589}

$$-12a\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+12a\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+24a\sinh^{-1}(ax)\text{PolyLog}($$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4/x^2,x]
```

```
[Out] -(ArcSinh[a*x]^4/x) - 8*a*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 12*a*Arc
Sinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]] + 12*a*ArcSinh[a*x]^2*PolyLog[2, E^
ArcSinh[a*x]] + 24*a*ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] - 24*a*ArcSin
h[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 24*a*PolyLog[4, -E^ArcSinh[a*x]] + 24*a
*PolyLog[4, E^ArcSinh[a*x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)^4}{x^2}\,dx &= -\frac{\sinh^{-1}(ax)^4}{x} + (4a)\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sinh^{-1}(ax)^4}{x} + (4a)\,\text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)\\ &= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - (12a)\,\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \text{si}\right.\\ &= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 12a\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 12a\\ &= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 12a\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 12a\\ &= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 12a\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 12a\\ &= -\frac{\sinh^{-1}(ax)^4}{x} - 8a\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 12a\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 12a\end{aligned}$$

Mathematica [A] time = 0.25, size = 161, normalized size = 1.34

$$\frac{1}{2}a\left(24\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{-\sinh^{-1}(ax)}\right) + 24\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right) + 48\sinh^{-1}(ax)\text{Li}_3\left(-e^{-\sinh^{-1}(ax)}\right) - 48\sin\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^4/x^2,x]
```

```
[Out] (a*(Pi^4 - 2*ArcSinh[a*x]^4 - (2*ArcSinh[a*x]^4)/(a*x) - 8*ArcSinh[a*x]^3*L
og[1 + E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24*A
rcSinh[a*x]^2*PolyLog[2, -E^(-ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog[2,
 E^ArcSinh[a*x]] + 48*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*ArcS
inh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])] + 4
8*PolyLog[4, E^ArcSinh[a*x]]))/2
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^4}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^4/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^4/x^2, x)
```

maple [A] time = 0.24, size = 217, normalized size = 1.81

$$-\frac{\text{arcsinh}\,(ax)^4}{x}+4a\,\text{arcsinh}\,(ax)^3\ln\left(1-ax-\sqrt{a^2x^2+1}\right)+12a\,\text{arcsinh}\,(ax)^2\,\text{polylog}\left(2, ax+\sqrt{a^2x^2+1}\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4/x^2,x)
[Out] -arcsinh(a*x)^4/x+4*a*arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))+12*a*arcsi
nh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-24*a*arcsinh(a*x)*polylog(3,a*x+
(a^2*x^2+1)^(1/2))+24*a*polylog(4,a*x+(a^2*x^2+1)^(1/2))-4*a*arcsinh(a*x)^3
*ln(a*x+(a^2*x^2+1)^(1/2)+1)-12*a*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)
^(1/2))+24*a*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-24*a*polylog(4,
-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^4}{x}+\int \frac{4\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^3x^4+ax^2+\left(a^2x^3+x\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="maxima")
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^4/x + integrate(4*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^4 + a*x^2 + (a^2*x^3 + x
)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{asinh}\,(a\,x)^4}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^4/x^2,x)
[Out] int(asinh(a*x)^4/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^4(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4/x**2,x)
```

```
[Out] Integral(asinh(a*x)**4/x**2, x)
```

3.40 $\int \frac{\sinh^{-1}(ax)^4}{x^3}\,dx$

Optimal. Leaf size=108

$$6a^2\sinh^{-1}(ax)\mathrm{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-3a^2\mathrm{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)-\frac{2a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{x}-2a^2\sinh^{-1}(ax)^3+6a^2\sinh^{-1}($$

```
[Out] -2*a^2*arcsinh(a*x)^3-1/2*arcsinh(a*x)^4/x^2+6*a^2*arcsinh(a*x)^2*ln(1-(a*x
+(a^2*x^2+1)^(1/2))^2)+6*a^2*arcsinh(a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))
^2)-3*a^2*polylog(3,(a*x+(a^2*x^2+1)^(1/2))^2)-2*a*arcsinh(a*x)^3*(a^2*x^2+
1)^(1/2)/x
```

Rubi [A] time = 0.21, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5661, 5723, 5659, 3716, 2190, 2531, 2282, 6589}

$$6a^2\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-3a^2\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{2a\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{x}-2a^2\sinh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4/x^3,x]
```

```
[Out] -2*a^2*ArcSinh[a*x]^3 - (2*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x - ArcSinh[
a*x]^4/(2*x^2) + 6*a^2*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 6*a^2*A
rcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*a^2*PolyLog[3, E^(2*ArcSinh[
a*x])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax)^4}{x^3}\,dx &= -\frac{\sinh^{-1}(ax)^4}{2x^2} + (2a)\int \frac{\sinh^{-1}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + \left(6a^2\right)\int \frac{\sinh^{-1}(ax)^2}{x}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + \left(6a^2\right)\text{Subst}\left(\int x^2\coth(x)\,dx, x, \sinh^{-1}(ax)\right) \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} - \left(12a^2\right)\text{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, \right. \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + 6a^2\sinh^{-1}(ax)^2\log\left(1-e^{2\,\mathrm{s}}\right. \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + 6a^2\sinh^{-1}(ax)^2\log\left(1-e^{2\,\mathrm{s}}\right. \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + 6a^2\sinh^{-1}(ax)^2\log\left(1-e^{2\,\mathrm{s}}\right. \\
&= -2a^2\sinh^{-1}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}{x} - \frac{\sinh^{-1}(ax)^4}{2x^2} + 6a^2\sinh^{-1}(ax)^2\log\left(1-e^{2\,\mathrm{s}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.28, size = 113, normalized size = 1.05

$$
-\frac{\sinh^{-1}(ax)^4}{2x^2}+\frac{1}{4}a^2\left(-\frac{8\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^3}{ax}+24\sinh^{-1}(ax)\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)-12\text{Li}_3\left(e^{2\sinh^{-1}(ax)}\right)-8\sinh^{-1}(\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^4/x^3,x]
```

```
[Out] -1/2*ArcSinh[a*x]^4/x^2 + (a^2*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x]^3)/(a*x) + 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])]
 + 24*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*ArcS
inh[a*x])]))/4
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arsinh}(ax)^4}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.37, size = 208, normalized size = 1.93

$$-2a^2 \operatorname{arcsinh}(ax)^3 - \frac{2a \operatorname{arcsinh}(ax)^3 \sqrt{a^2x^2+1}}{x} - \frac{\operatorname{arcsinh}(ax)^4}{2x^2} + 6a^2 \operatorname{arcsinh}(ax)^2 \ln\left(1 - ax - \sqrt{a^2x^2+1}\right) + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4/x^3,x)
```

```
[Out] -2*a^2*arcsinh(a*x)^3-2*a*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x-1/2*arcsinh(a*
x)^4/x^2+6*a^2*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+12*a^2*arcsinh(a*
x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-12*a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2))
+6*a^2*arcsinh(a*x)^2*ln(a*x+(a^2*x^2+1)^(1/2)+1)+12*a^2*arcsinh(a*x)*polyl
og(2,-a*x-(a^2*x^2+1)^(1/2))-12*a^2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax + \sqrt{a^2x^2+1}\right)^4}{2\,x^2} + \int \frac{2\left(a^3x^2 + \sqrt{a^2x^2+1}\,a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)^3}{a^3x^5 + ax^3 + \left(a^2x^4 + x^2\right)\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + sqrt(a^2*x^2 + 1))^4/x^2 + integrate(2*(a^3*x^2 + sqrt(a^2*x
^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^5 + a*x^3 + (a^2*x
^4 + x^2)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x)^4}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^4/x^3,x)
```

```
[Out] int(asinh(a*x)^4/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^4(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4/x**3,x)
```

```
[Out] Integral(asinh(a*x)**4/x**3, x)
```

3.41 $\int \frac{\sinh^{-1}(ax)^4}{x^4}\,dx$

Optimal. Leaf size=223

$$2a^3\sinh^{-1}(ax)^2\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-2a^3\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)-4a^3\sinh^{-1}(ax)\text{Li}_3\left(-e^{\sinh^{-1}(ax)}\right)+4a^3\sinh^{-1}$$

```
[Out] -2*a^2*arcsinh(a*x)^2/x-1/3*arcsinh(a*x)^4/x^3-8*a^3*arcsinh(a*x)*arctanh(a
*x+(a^2*x^2+1)^(1/2))+4/3*a^3*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))
-4*a^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*a^3*arcsinh(a*x)^2*polylog(2,-a*
x-(a^2*x^2+1)^(1/2))+4*a^3*polylog(2,a*x+(a^2*x^2+1)^(1/2))-2*a^3*arcsinh(a
*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-4*a^3*arcsinh(a*x)*polylog(3,-a*x-(a
^2*x^2+1)^(1/2))+4*a^3*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+4*a^3*
polylog(4,-a*x-(a^2*x^2+1)^(1/2))-4*a^3*polylog(4,a*x+(a^2*x^2+1)^(1/2))-2/
3*a*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.39, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5661, 5747, 5760, 4182, 2531, 6609, 2282, 6589, 2279, 2391}

$$2a^3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-2a^3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-4a^3\sinh^{-1}(ax)\text{PolyLog}\left(3,\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^4/x^4,x]
```

```
[Out] (-2*a^2*ArcSinh[a*x]^2)/x - (2*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*x^2)
- ArcSinh[a*x]^4/(3*x^3) - 8*a^3*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + (4*
a^3*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]])/3 - 4*a^3*PolyLog[2, -E^ArcSinh
[a*x]] + 2*a^3*ArcSinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]] + 4*a^3*PolyLog[2
, E^ArcSinh[a*x]] - 2*a^3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] - 4*a^3
*ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] + 4*a^3*ArcSinh[a*x]*PolyLog[3, E
^ArcSinh[a*x]] + 4*a^3*PolyLog[4, -E^ArcSinh[a*x]] - 4*a^3*PolyLog[4, E^Arc
Sinh[a*x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^4}{x^4}\,dx = -\frac{\sinh^{-1}(ax)^4}{3x^3} + \frac{1}{3}(4a)\int \frac{\sinh^{-1}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} + \left(2a^2\right)\int \frac{\sinh^{-1}(ax)^2}{x^2}\,dx - \frac{1}{3}\left(2a^3\right)\int \frac{\sinh^{-1}}{x\sqrt{1-}}$$
$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - \frac{1}{3}\left(2a^3\right)\text{Subst}\left(\int x^3\text{csch}\right.$$
$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} + \frac{4}{3}a^3\sinh^{-1}(ax)^3\tanh^{-1}$$
$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - 8a^3\sinh^{-1}(ax)\tanh^{-1}\left(e\right.$$
$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - 8a^3\sinh^{-1}(ax)\tanh^{-1}\left(e\right.$$
$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - 8a^3\sinh^{-1}(ax)\tanh^{-1}\left(e\right.$$
$$= -\frac{2a^2\sinh^{-1}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3x^2} - \frac{\sinh^{-1}(ax)^4}{3x^3} - 8a^3\sinh^{-1}(ax)\tanh^{-1}\left(e\right.$$

Mathematica [A] time = 2.64, size = 355, normalized size = 1.59

$$\frac{1}{24}a^3\left(-\frac{8\sinh^4\left(\frac{1}{2}\sinh^{-1}(ax)\right)\sinh^{-1}(ax)^4}{a^3x^3} - 48\sinh^{-1}(ax)^2\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right) - 96\sinh^{-1}(ax)\text{Li}_3\left(-e^{-\sinh^{-1}(ax)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^4/x^4,x]
```

```
[Out] (a^3*(-2*Pi^4 + 4*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*Coth[ArcSinh[a*x]/2] +
 2*ArcSinh[a*x]^4*Coth[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Csch[ArcSinh[a*x]
/2]^2 - (a*x*ArcSinh[a*x]^4*Csch[ArcSinh[a*x]/2]^4)/2 + 96*ArcSinh[a*x]*Log
[1 - E^(-ArcSinh[a*x])] - 96*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] + 16*A
rcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 16*ArcSinh[a*x]^3*Log[1 - E^ArcS
inh[a*x]] - 48*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^(-ArcSinh[a*x])] - 96*Po
lyLog[2, E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]]
- 96*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] + 96*ArcSinh[a*x]*PolyLog[
3, E^ArcSinh[a*x]] - 96*PolyLog[4, -E^(-ArcSinh[a*x])] - 96*PolyLog[4, E^Ar
cSinh[a*x]] - 4*ArcSinh[a*x]^3*Sech[ArcSinh[a*x]/2]^2 - (8*ArcSinh[a*x]^4*S
inh[ArcSinh[a*x]/2]^4)/(a^3*x^3) + 24*ArcSinh[a*x]^2*Tanh[ArcSinh[a*x]/2] -
 2*ArcSinh[a*x]^4*Tanh[ArcSinh[a*x]/2]))/24
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\left(ax\right)^4}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\left(ax\right)^4}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^4/x^4, x)
```

maple [A] time = 0.48, size = 372, normalized size = 1.67

$$-\frac{2a\,\mathrm{arcsinh}\,(ax)^3\sqrt{a^2x^2+1}}{3x^2}-\frac{2a^2\,\mathrm{arcsinh}\,(ax)^2}{x}-\frac{\mathrm{arcsinh}\,(ax)^4}{3x^3}-\frac{2a^3\,\mathrm{arcsinh}\,(ax)^3\ln\left(1-ax-\sqrt{a^2x^2+1}\right)}{3}-2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^4/x^4,x)
```

```
[Out] -2/3*a*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x^2-2*a^2*arcsinh(a*x)^2/x-1/3*arcs
inh(a*x)^4/x^3-2/3*a^3*arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))-2*a^3*arc
sinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+4*a^3*arcsinh(a*x)*polylog(3,a
*x+(a^2*x^2+1)^(1/2))-4*a^3*polylog(4,a*x+(a^2*x^2+1)^(1/2))+2/3*a^3*arcsin
h(a*x)^3*ln(a*x+(a^2*x^2+1)^(1/2)+1)+2*a^3*arcsinh(a*x)^2*polylog(2,-a*x-(a
^2*x^2+1)^(1/2))-4*a^3*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+4*a^3
*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+4*a^3*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^
(1/2))+4*a^3*polylog(2,a*x+(a^2*x^2+1)^(1/2))-4*a^3*arcsinh(a*x)*ln(a*x+(a^
2*x^2+1)^(1/2)+1)-4*a^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^4}{3\,x^3}+\int\frac{4\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{3\left(a^3x^6+ax^4+\left(a^2x^5+x^3\right)\sqrt{a^2x^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^4/x^3 + integrate(4/3*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^6 + a*x^4 + (a^2
*x^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\mathrm{asinh}\,(a\,x)^4}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^4/x^4,x)
```

```
[Out] int(asinh(a*x)^4/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{asinh}^4\,(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**4/x**4,x)
```

```
[Out] Integral(asinh(a*x)**4/x**4, x)
```

3.42 $\int \frac{x^6}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$-\frac{5\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{9\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{5\mathrm{Chi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{\mathrm{Chi}\left(7\sinh^{-1}(ax)\right)}{64a^7}$$

```
[Out] -5/64*Chi(arcsinh(a*x))/a^7+9/64*Chi(3*arcsinh(a*x))/a^7-5/64*Chi(5*arcsinh
(a*x))/a^7+1/64*Chi(7*arcsinh(a*x))/a^7
```

Rubi [A] time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5669, 5448, 3301}

$$-\frac{5\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{9\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{5\mathrm{Chi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{\mathrm{Chi}\left(7\sinh^{-1}(ax)\right)}{64a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/ArcSinh[a*x],x]
```

```
[Out] (-5*CoshIntegral[ArcSinh[a*x]])/(64*a^7) + (9*CoshIntegral[3*ArcSinh[a*x]])
/(64*a^7) - (5*CoshIntegral[5*ArcSinh[a*x]])/(64*a^7) + CoshIntegral[7*ArcS
inh[a*x]]/(64*a^7)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^6}{\sinh^{-1}(ax)}\,dx=\frac{\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^7}$$

$$=\frac{\mathrm{Subst}\left(\int \left(-\frac{5\cosh(x)}{64x}+\frac{9\cosh(3x)}{64x}-\frac{5\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^7}$$

$$=\frac{\mathrm{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{64a^7}-\frac{5\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{64a^7}-\frac{5\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$=-\frac{5\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{9\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{5\mathrm{Chi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{\mathrm{Chi}\left(7\sinh^{-1}(ax)\right)}{64a^7}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.73

$$\frac{-5\text{Chi}\left(\sinh^{-1}(ax)\right)+9\text{Chi}\left(3\sinh^{-1}(ax)\right)-5\text{Chi}\left(5\sinh^{-1}(ax)\right)+\text{Chi}\left(7\sinh^{-1}(ax)\right)}{64a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/ArcSinh[a*x],x]
```

```
[Out] (-5*CoshIntegral[ArcSinh[a*x]] + 9*CoshIntegral[3*ArcSinh[a*x]] - 5*CoshInt
egral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]])/(64*a^7)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^6/arcsinh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^6/arcsinh(a*x), x)
```

maple [A] time = 0.20, size = 40, normalized size = 0.73

$$\frac{-\frac{5\text{X}(\text{arcsinh}(ax))}{64}+\frac{9\text{X}(3\,\text{arcsinh}(ax))}{64}-\frac{5\text{X}(5\,\text{arcsinh}(ax))}{64}+\frac{\text{X}(7\,\text{arcsinh}(ax))}{64}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/arcsinh(a*x),x)
```

```
[Out] 1/a^7*(-5/64*Chi(arcsinh(a*x))+9/64*Chi(3*arcsinh(a*x))-5/64*Chi(5*arcsinh(
a*x))+1/64*Chi(7*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\text{asinh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/asinh(a*x),x)
[Out] int(x^6/asinh(a*x), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/asinh(a*x),x)
[Out] Integral(x**6/asinh(a*x), x)
```

3.43 $\int \frac{x^5}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=43

$$\frac{5\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{32a^6}-\frac{\mathrm{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^6}+\frac{\mathrm{Shi}\left(6\sinh^{-1}(ax)\right)}{32a^6}$$

```
[Out] 5/32*Shi(2*arcsinh(a*x))/a^6-1/8*Shi(4*arcsinh(a*x))/a^6+1/32*Shi(6*arcsinh
(a*x))/a^6
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5669, 5448, 3298}

$$\frac{5\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{32a^6}-\frac{\mathrm{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^6}+\frac{\mathrm{Shi}\left(6\sinh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/ArcSinh[a*x],x]
```

```
[Out] (5*SinhIntegral[2*ArcSinh[a*x]])/(32*a^6) - SinhIntegral[4*ArcSinh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcSinh[a*x]]/(32*a^6)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sinh^{-1}(ax)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^6}\\
&= \frac{\mathrm{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x}-\frac{\sinh(4x)}{8x}+\frac{\sinh(6x)}{32x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^6}\\
&= \frac{\mathrm{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{32a^6}-\frac{\mathrm{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{8a^6}+\frac{5\,\mathrm{Subst}\left(\int \frac{\sinh(2}{x}\right.}{3}\\
&= \frac{5\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{32a^6}-\frac{\mathrm{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^6}+\frac{\mathrm{Shi}\left(6\sinh^{-1}(ax)\right)}{32a^6}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 33, normalized size = 0.77

$$\frac{5\text{Shi}\left(2\sinh^{-1}(ax)\right)-4\text{Shi}\left(4\sinh^{-1}(ax)\right)+\text{Shi}\left(6\sinh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/ArcSinh[a*x],x]
```

```
[Out] (5*SinhIntegral[2*ArcSinh[a*x]] - 4*SinhIntegral[4*ArcSinh[a*x]] + SinhInte
gral[6*ArcSinh[a*x]])/(32*a^6)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\text{arsinh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^5/arcsinh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.19, size = 33, normalized size = 0.77

$$\frac{\frac{5\,\text{Shi}(2\,\text{arcsinh}(ax))}{32}-\frac{\text{Shi}(4\,\text{arcsinh}(ax))}{8}+\frac{\text{Shi}(6\,\text{arcsinh}(ax))}{32}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/arcsinh(a*x),x)
```

```
[Out] 1/a^6*(5/32*Shi(2*arcsinh(a*x))-1/8*Shi(4*arcsinh(a*x))+1/32*Shi(6*arcsinh(
a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\text{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5}{\text{asinh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/asinh(a*x),x)
```

```
[Out] int(x^5/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/asinh(a*x),x)
```

```
[Out] Integral(x**5/asinh(a*x), x)
```

3.44 $\int \frac{x^4}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^5}-\frac{3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a^5}+\frac{\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

```
[Out] 1/8*Chi(arcsinh(a*x))/a^5-3/16*Chi(3*arcsinh(a*x))/a^5+1/16*Chi(5*arcsinh(a
*x))/a^5
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5669, 5448, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^5}-\frac{3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a^5}+\frac{\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x],x]
```

```
[Out] CoshIntegral[ArcSinh[a*x]]/(8*a^5) - (3*CoshIntegral[3*ArcSinh[a*x]])/(16*a
^5) + CoshIntegral[5*ArcSinh[a*x]]/(16*a^5)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\cosh(x)}{8x}-\frac{3\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{16a^5}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5}-\frac{3\,\text{Subst}\left(\int \frac{\text{cos}}{}\right.}{}\\
&= \frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^5}-\frac{3\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a^5}+\frac{\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.76

$$\frac{2\text{Chi}\left(\sinh^{-1}(ax)\right)-3\text{Chi}\left(3\sinh^{-1}(ax)\right)+\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcSinh[a*x],x]
```

```
[Out] (2*CoshIntegral[ArcSinh[a*x]] - 3*CoshIntegral[3*ArcSinh[a*x]] + CoshIntegr
al[5*ArcSinh[a*x]])/(16*a^5)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x), x)
```

maple [A] time = 0.16, size = 31, normalized size = 0.76

$$\frac{\frac{\text{X}(\text{arcsinh}(ax))}{8}-\frac{3\text{X}(3\,\text{arcsinh}(ax))}{16}+\frac{\text{X}(5\,\text{arcsinh}(ax))}{16}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x),x)
```

```
[Out] 1/a^5*(1/8*Chi(arcsinh(a*x))-3/16*Chi(3*arcsinh(a*x))+1/16*Chi(5*arcsinh(a*
x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\text{asinh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asinh(a*x),x)
```

```
[Out] int(x^4/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x),x)
```

```
[Out] Integral(x**4/asinh(a*x), x)
```

3.45 $\int \frac{x^3}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^4}-\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{4a^4}$$

```
[Out] -1/4*Shi(2*arcsinh(a*x))/a^4+1/8*Shi(4*arcsinh(a*x))/a^4
```

Rubi [A] time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5669, 5448, 3298}

$$\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^4}-\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x],x]
```

```
[Out] -SinhIntegral[2*ArcSinh[a*x]]/(4*a^4) + SinhIntegral[4*ArcSinh[a*x]]/(8*a^4
)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{8a^4}-\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{4a^4}\\
&= -\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{4a^4}+\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{8a^4}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 24, normalized size = 0.83

$$\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)-2\text{Shi}\left(2\sinh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcSinh[a*x],x]
[Out] (-2*SinhIntegral[2*ArcSinh[a*x]] + SinhIntegral[4*ArcSinh[a*x]])/(8*a^4)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x^3/arcsinh(a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.16, size = 24, normalized size = 0.83

$$\frac{-\frac{\text{Shi}(2\,\text{arcsinh}(ax))}{4}+\frac{\text{Shi}(4\,\text{arcsinh}(ax))}{8}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x),x)
[Out] 1/a^4*(-1/4*Shi(2*arcsinh(a*x))+1/8*Shi(4*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(x^3/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x^3}{\text{asinh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asinh(a*x),x)
```

```
[Out] int(x^3/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x),x)
```

```
[Out] Integral(x**3/asinh(a*x), x)
```

3.46 $\int \frac{x^2}{\sinh^{-1}(ax)} \, dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a^3}-\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a^3}$$

```
[Out] -1/4*Chi(arcsinh(a*x))/a^3+1/4*Chi(3*arcsinh(a*x))/a^3
```

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5669, 5448, 3301}

$$\frac{\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a^3}-\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x],x]
```

```
[Out] -CoshIntegral[ArcSinh[a*x]]/(4*a^3) + CoshIntegral[3*ArcSinh[a*x]]/(4*a^3)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sinh^{-1}(ax)} \, dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x} \, dx, x, \sinh^{-1}(ax)\right)}{a^3}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right) dx, x, \sinh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x} \, dx, x, \sinh^{-1}(ax)\right)}{4a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x} \, dx, x, \sinh^{-1}(ax)\right)}{4a^3}\\
&= -\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a^3}+\frac{\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)-\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a*x],x]
[Out] (-CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2}{\mathrm{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x^2/arcsinh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(x^2/arcsinh(a*x), x)
```

maple [A] time = 0.13, size = 22, normalized size = 0.81

$$\frac{-\frac{\mathrm{X}(\mathrm{arcsinh}(ax))}{4}+\frac{\mathrm{X}(3\,\mathrm{arcsinh}(ax))}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x),x)
[Out] 1/a^3*(-1/4*Chi(arcsinh(a*x))+1/4*Chi(3*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(x^2/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\mathrm{asinh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a*x),x)
```

```
[Out] int(x^2/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x),x)
[Out] Integral(x**2/asinh(a*x), x)
```

3.47 $\int \frac{x}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=14

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^2}$$

```
[Out] 1/2*Shi(2*arcsinh(a*x))/a^2
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5669, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x],x]
```

```
[Out] SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{2a^2}\\
&= \frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a*x],x]
[Out] SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\text{arsinh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x/arcsinh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(x/arcsinh(a*x), x)
```

maple [A] time = 0.16, size = 13, normalized size = 0.93

$$\frac{\text{Shi}\,(2\,\text{arcsinh}\,(ax))}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x),x)
[Out] 1/2*Shi(2*arcsinh(a*x))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(x/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\text{asinh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a*x),x)
[Out] int(x/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x),x)
```

```
[Out] Integral(x/asinh(a*x), x)
```

3.48 $\int \frac{1}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{a}$$

[Out] Chi(arcsinh(a*x))/a

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5657, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a*x]^(-1),x]

[Out] CoshIntegral[ArcSinh[a*x]]/a

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSinh[a*x]^(-1),x]

[Out] CoshIntegral[ArcSinh[a*x]]/a

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\text{arsinh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/arcsinh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/arcsinh(a*x), x)
```

maple [A] time = 0.10, size = 10, normalized size = 1.11

$$\frac{\mathrm{X}(\operatorname{arcsinh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x),x)
```

```
[Out] Chi(arcsinh(a*x))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/arcsinh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\operatorname{asinh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a*x),x)
```

```
[Out] int(1/asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x),x)
```

```
[Out] Integral(1/asinh(a*x), x)
```

3.49 $\int \frac{1}{x\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]),x]
[Out] Defer[Int][1/(x*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\sinh^{-1}(ax)}\,dx = \int \frac{1}{x\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]),x]
[Out] Integrate[1/(x*ArcSinh[a*x]), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\,\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x),x)
[Out] int(1/x/arcsinh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{asinh}(a\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)),x)
[Out] int(1/(x*asinh(a*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x),x)
[Out] Integral(1/(x*asinh(a*x)), x)
```

3.50 $\int \frac{1}{x^2 \sinh^{-1}(ax)} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \sinh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \sinh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \sinh^{-1}(ax)} \, dx = \int \frac{1}{x^2 \sinh^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sinh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcSinh[a*x]),x]
```

```
[Out] Integrate[1/(x^2*ArcSinh[a*x]), x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*arcsinh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arsinh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)), x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x),x)
[Out] int(1/x^2/arcsinh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/(x^2*arcsinh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{asinh}(a\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*asinh(a*x)),x)
[Out] int(1/(x^2*asinh(a*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{asinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x),x)
[Out] Integral(1/(x**2*asinh(a*x)), x)
```

3.51 $\int \frac{x^6}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{5\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{27\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{25\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{7\text{Shi}\left(7\sinh^{-1}(ax)\right)}{64a^7}-\frac{x^6\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -5/64*Shi(arcsinh(a*x))/a^7+27/64*Shi(3*arcsinh(a*x))/a^7-25/64*Shi(5*arcsi
nh(a*x))/a^7+7/64*Shi(7*arcsinh(a*x))/a^7-x^6*(a^2*x^2+1)^(1/2)/a/arcsinh(a
*x)
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5665, 3298}

$$-\frac{5\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{27\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{25\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{7\text{Shi}\left(7\sinh^{-1}(ax)\right)}{64a^7}-\frac{x^6\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^6*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - (5*SinhIntegral[ArcSinh[a*x]]
)/(64*a^7) + (27*SinhIntegral[3*ArcSinh[a*x]])/(64*a^7) - (25*SinhIntegral[
5*ArcSinh[a*x]])/(64*a^7) + (7*SinhIntegral[7*ArcSinh[a*x]])/(64*a^7)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sinh^{-1}(ax)^2}\,dx &= -\frac{x^6\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\text{Subst}\left(\int \left(-\frac{5\sinh(x)}{64x}+\frac{27\sinh(3x)}{64x}-\frac{25\sinh(5x)}{64x}+\frac{7\sinh(7x)}{64x}\right)dx,x,\sinh^{-1}(a\right.}{a^7}\\ &= -\frac{x^6\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}-\frac{5\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{64a^7}+\frac{7\,\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx,x,\sinh^{-1}(\right.}{64a^7}\\ &= -\frac{x^6\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}-\frac{5\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a^7}+\frac{27\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a^7}-\frac{25\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a^7}+\frac{7}{}\end{aligned}$$

Mathematica [A] time = 0.29, size = 85, normalized size = 1.04

$$-\frac{64a^6x^6\sqrt{a^2x^2+1}+5\sinh^{-1}(ax)\text{Shi}\left(\sinh^{-1}(ax)\right)-27\sinh^{-1}(ax)\text{Shi}\left(3\sinh^{-1}(ax)\right)+25\sinh^{-1}(ax)\text{Shi}\left(5\right.}{64a^7\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/ArcSinh[a*x]^2,x]
```

```
[Out] -1/64*(64*a^6*x^6*Sqrt[1 + a^2*x^2] + 5*ArcSinh[a*x]*SinhIntegral[ArcSinh[a
*x]] - 27*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 25*ArcSinh[a*x]*SinhI
ntegral[5*ArcSinh[a*x]] - 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]])/(a^7
*ArcSinh[a*x])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6/arcsinh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{arsinh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/arcsinh(a*x)^2, x)
```

maple [A] time = 0.21, size = 104, normalized size = 1.27

$$\frac{\frac{5\sqrt{a^2x^2+1}}{64\operatorname{arcsinh}(ax)} - \frac{5\operatorname{Shi}(\operatorname{arcsinh}(ax))}{64} - \frac{9\cosh(3\operatorname{arcsinh}(ax))}{64\operatorname{arcsinh}(ax)} + \frac{27\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{64} + \frac{5\cosh(5\operatorname{arcsinh}(ax))}{64\operatorname{arcsinh}(ax)} - \frac{25\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{64} - \frac{\cosh(\cdots}{64\cdots}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^7*(5/64/arcsinh(a*x)*(a^2*x^2+1)^(1/2)-5/64*Shi(arcsinh(a*x))-9/64/arcs
inh(a*x)*cosh(3*arcsinh(a*x))+27/64*Shi(3*arcsinh(a*x))+5/64/arcsinh(a*x)*c
osh(5*arcsinh(a*x))-25/64*Shi(5*arcsinh(a*x))-1/64/arcsinh(a*x)*cosh(7*arcs
inh(a*x))+7/64*Shi(7*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^9 + ax^7 + \left(a^2x^8 + x^6\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}\,a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{7\,a^5x^{10} + 14\,a^3x^8 + 7\,ax^6 + \left(7\,a^3x^8 + 5\,ax^6\right)\left(a^2x^2+1\right) + \cdots}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^9 + a*x^7 + (a^2*x^8 + x^6)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((7*a^5*x^10
+ 14*a^3*x^8 + 7*a*x^6 + (7*a^3*x^8 + 5*a*x^6)*(a^2*x^2 + 1) + (14*a^4*x^9
+ 19*a^2*x^7 + 6*x^5)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2
+ 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2
*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\operatorname{asinh}(a\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/asinh(a*x)^2,x)
```

```
[Out] int(x^6/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/asinh(a*x)**2,x)
```

```
[Out] Integral(x**6/asinh(a*x)**2, x)
```

3.52 $\int \frac{x^5}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5\text{Chi}\left(2\sinh^{-1}(ax)\right)}{16a^6}-\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^6}+\frac{3\text{Chi}\left(6\sinh^{-1}(ax)\right)}{16a^6}-\frac{x^5\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] 5/16*Chi(2*arcsinh(a*x))/a^6-1/2*Chi(4*arcsinh(a*x))/a^6+3/16*Chi(6*arcsinh
(a*x))/a^6-x^5*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5665, 3301}

$$\frac{5\text{Chi}\left(2\sinh^{-1}(ax)\right)}{16a^6}-\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^6}+\frac{3\text{Chi}\left(6\sinh^{-1}(ax)\right)}{16a^6}-\frac{x^5\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^5*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + (5*CoshIntegral[2*ArcSinh[a*x
]])/(16*a^6) - CoshIntegral[4*ArcSinh[a*x]]/(2*a^6) + (3*CoshIntegral[6*Arc
Sinh[a*x]])/(16*a^6)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sinh^{-1}(ax)^2}\,dx &= -\frac{x^5\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\text{Subst}\left(\int \left(\frac{5\cosh(2x)}{16x}-\frac{\cosh(4x)}{2x}+\frac{3\cosh(6x)}{16x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^6}\\ &= -\frac{x^5\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{3\,\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{16a^6}+\frac{5\,\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx,x,\sinh^{-1}(a\right.}{16a^6}\\ &= -\frac{x^5\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{5\text{Chi}\left(2\sinh^{-1}(ax)\right)}{16a^6}-\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^6}+\frac{3\text{Chi}\left(6\sinh^{-1}(ax)\right)}{16a^6}\end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 1.11

$$-\frac{-10\sinh^{-1}(ax)\text{Chi}\left(2\sinh^{-1}(ax)\right)+16\sinh^{-1}(ax)\text{Chi}\left(4\sinh^{-1}(ax)\right)-6\sinh^{-1}(ax)\text{Chi}\left(6\sinh^{-1}(ax)\right)+5\sin}{32a^6\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/ArcSinh[a*x]^2,x]
```

```
[Out] -1/32*(-10*ArcSinh[a*x]*CoshIntegral[2*ArcSinh[a*x]] + 16*ArcSinh[a*x]*Cosh
Integral[4*ArcSinh[a*x]] - 6*ArcSinh[a*x]*CoshIntegral[6*ArcSinh[a*x]] + 5*
Sinh[2*ArcSinh[a*x]] - 4*Sinh[4*ArcSinh[a*x]] + Sinh[6*ArcSinh[a*x]])/(a^6*
ArcSinh[a*x])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5/arcsinh(a*x)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.20, size = 78, normalized size = 1.11

$$\frac{-\frac{5\sinh(2\operatorname{arcsinh}(ax))}{32\operatorname{arcsinh}(ax)}+\frac{5X(2\operatorname{arcsinh}(ax))}{16}+\frac{\sinh(4\operatorname{arcsinh}(ax))}{8\operatorname{arcsinh}(ax)}-\frac{X(4\operatorname{arcsinh}(ax))}{2}-\frac{\sinh(6\operatorname{arcsinh}(ax))}{32\operatorname{arcsinh}(ax)}+\frac{3X(6\operatorname{arcsinh}(ax))}{16}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^6*(-5/32/arcsinh(a*x)*sinh(2*arcsinh(a*x))+5/16*Chi(2*arcsinh(a*x))+1/8
/arcsinh(a*x)*sinh(4*arcsinh(a*x))-1/2*Chi(4*arcsinh(a*x))-1/32/arcsinh(a*x
)*sinh(6*arcsinh(a*x))+3/16*Chi(6*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^8+ax^6+\left(a^2x^7+x^5\right)\sqrt{a^2x^2+1}}{\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}+\int\frac{6\,a^5x^9+12\,a^3x^7+6\,ax^5+2\left(3\,a^3x^7+2\,ax^5\right)\left(a^2x^2+1\right.}{\left(a^5x^4+\left(a^2x^2+1\right)a^3x^2+2\,a^3x^2+2\left(a^4x^3+a^2x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^8 + a*x^6 + (a^2*x^7 + x^5)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((6*a^5*x^9 +
 12*a^3*x^7 + 6*a*x^5 + 2*(3*a^3*x^7 + 2*a*x^5)*(a^2*x^2 + 1) + (12*a^4*x^8
 + 16*a^2*x^6 + 5*x^4)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2
 + 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^
2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\operatorname{asinh}(a\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/asinh(a*x)^2,x)
```

```
[Out] int(x^5/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/asinh(a*x)**2,x)
```

```
[Out] Integral(x**5/asinh(a*x)**2, x)
```

3.53 $\int \frac{x^4}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=68

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a^5}-\frac{9\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a^5}+\frac{5\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a^5}-\frac{x^4\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] 1/8*Shi(arcsinh(a*x))/a^5-9/16*Shi(3*arcsinh(a*x))/a^5+5/16*Shi(5*arcsinh(a
*x))/a^5-x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5665, 3298}

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a^5}-\frac{9\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a^5}+\frac{5\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a^5}-\frac{x^4\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/(8
*a^5) - (9*SinhIntegral[3*ArcSinh[a*x]])/(16*a^5) + (5*SinhIntegral[5*ArcSi
nh[a*x]])/(16*a^5)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sinh^{-1}(ax)^2}\,dx &= -\frac{x^4\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\text{Subst}\left(\int\left(\frac{\sinh(x)}{8x}-\frac{9\sinh(3x)}{16x}+\frac{5\sinh(5x)}{16x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^5}\\ &= -\frac{x^4\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\text{Subst}\left(\int\frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{8a^5}+\frac{5\,\text{Subst}\left(\int\frac{\sinh(5x)}{x}\,dx,x,\sinh^{-1}(a\right.}{16a^5}\\ &= -\frac{x^4\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{8a^5}-\frac{9\text{Shi}\left(3\sinh^{-1}(ax)\right)}{16a^5}+\frac{5\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a^5}\end{aligned}$$

Mathematica [A] time = 0.21, size = 60, normalized size = 0.88

$$\frac{-\frac{16a^4x^4\sqrt{a^2x^2+1}}{\sinh^{-1}(ax)}+2\text{Shi}\left(\sinh^{-1}(ax)\right)-9\text{Shi}\left(3\sinh^{-1}(ax)\right)+5\text{Shi}\left(5\sinh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcSinh[a*x]^2,x]
```

```
[Out] ((-16*a^4*x^4*Sqrt[1 + a^2*x^2])/ArcSinh[a*x] + 2*SinhIntegral[ArcSinh[a*x]
] - 9*SinhIntegral[3*ArcSinh[a*x]] + 5*SinhIntegral[5*ArcSinh[a*x]])/(16*a^
5)
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^2, x)
```

maple [A] time = 0.17, size = 80, normalized size = 1.18

$$\frac{-\frac{\sqrt{a^2x^2+1}}{8\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{8}+\frac{3\cosh(3\operatorname{arcsinh}(ax))}{16\operatorname{arcsinh}(ax)}-\frac{9\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{16}-\frac{\cosh(5\operatorname{arcsinh}(ax))}{16\operatorname{arcsinh}(ax)}+\frac{5\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{16}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^5*(-1/8/arcsinh(a*x)*(a^2*x^2+1)^(1/2)+1/8*Shi(arcsinh(a*x))+3/16/arcsi
nh(a*x)*cosh(3*arcsinh(a*x))-9/16*Shi(3*arcsinh(a*x))-1/16/arcsinh(a*x)*cos
h(5*arcsinh(a*x))+5/16*Shi(5*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^7 + ax^5 + \left(a^2x^6 + x^4\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}\,a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{5\,a^5x^8 + 10\,a^3x^6 + 5\,ax^4 + \left(5\,a^3x^6 + 3\,ax^4\right)\left(a^2x^2+1\right) + \left(}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^7 + a*x^5 + (a^2*x^6 + x^4)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^5*x^8 +
 10*a^3*x^6 + 5*a*x^4 + (5*a^3*x^6 + 3*a*x^4)*(a^2*x^2 + 1) + (10*a^4*x^7 +
 13*a^2*x^5 + 4*x^3)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 +
 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*
x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{asinh}(a\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asinh(a*x)^2,x)
```

```
[Out] int(x^4/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**2,x)
```

```
[Out] Integral(x**4/asinh(a*x)**2, x)
```

3.54 $\int \frac{x^3}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -1/2*Chi(2*arcsinh(a*x))/a^4+1/2*Chi(4*arcsinh(a*x))/a^4-x^3*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5665, 3301}

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^4}-\frac{x^3\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - CoshIntegral[2*ArcSinh[a*x]]/
(2*a^4) + CoshIntegral[4*ArcSinh[a*x]]/(2*a^4)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sinh^{-1}(ax)^2}\,dx &= -\frac{x^3\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\text{Subst}\left(\int\left(-\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{2x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^4}\\
&= -\frac{x^3\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}-\frac{\text{Subst}\left(\int\frac{\cosh(2x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{2a^4}+\frac{\text{Subst}\left(\int\frac{\cosh(4x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{2a^4}\\
&= -\frac{x^3\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$-\frac{4\sinh^{-1}(ax)\text{Chi}\left(2\sinh^{-1}(ax)\right)-4\sinh^{-1}(ax)\text{Chi}\left(4\sinh^{-1}(ax)\right)-2\sinh\left(2\sinh^{-1}(ax)\right)+\sinh\left(4\sinh^{-1}(ax)\right)}{8a^4\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcSinh[a*x]^2,x]
```

```
[Out] -1/8*(4*ArcSinh[a*x]*CoshIntegral[2*ArcSinh[a*x]] - 4*ArcSinh[a*x]*CoshInte
gral[4*ArcSinh[a*x]] - 2*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]])/(a^4*
ArcSinh[a*x])
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\text{arsinh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.16, size = 54, normalized size = 0.96

$$\frac{\frac{\sinh(2\,\text{arcsinh}(ax))}{4\,\text{arcsinh}(ax)} - \frac{\text{X}(2\,\text{arcsinh}(ax))}{2} - \frac{\sinh(4\,\text{arcsinh}(ax))}{8\,\text{arcsinh}(ax)} + \frac{\text{X}(4\,\text{arcsinh}(ax))}{2}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^4*(1/4/arcsinh(a*x)*sinh(2*arcsinh(a*x))-1/2*Chi(2*arcsinh(a*x))-1/8/ar
csinh(a*x)*sinh(4*arcsinh(a*x))+1/2*Chi(4*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^6 + ax^4 + \left(a^2x^5 + x^3\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}\,a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{4\,a^5x^7 + 8\,a^3x^5 + 4\,ax^3 + 2\left(2\,a^3x^5 + ax^3\right)\left(a^2x^2+1\right)}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^6 + a*x^4 + (a^2*x^5 + x^3)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((4*a^5*x^7 +
 8*a^3*x^5 + 4*a*x^3 + 2*(2*a^3*x^5 + a*x^3)*(a^2*x^2 + 1) + (8*a^4*x^6 + 1
0*a^2*x^4 + 3*x^2)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2
*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^
2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\text{asinh}\,(a\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asinh(a*x)^2,x)
```

```
[Out] int(x^3/asinh(a*x)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**2,x)
```

```
[Out] Integral(x**3/asinh(a*x)**2, x)
```

3.55 $\int \frac{x^2}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\operatorname{Shi}\left(\sinh^{-1}(ax)\right)}{4a^3}+\frac{3\operatorname{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] -1/4*Shi(arcsinh(a*x))/a^3+3/4*Shi(3*arcsinh(a*x))/a^3-x^2*(a^2*x^2+1)^(1/2
)/a/arcsinh(a*x)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5665, 3298}

$$-\frac{\operatorname{Shi}\left(\sinh^{-1}(ax)\right)}{4a^3}+\frac{3\operatorname{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - SinhIntegral[ArcSinh[a*x]]/(4
*a^3) + (3*SinhIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sinh^{-1}(ax)^2}\,dx &= -\frac{x^2\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\operatorname{Subst}\left(\int \left(-\frac{\sinh(x)}{4x}+\frac{3\sinh(3x)}{4x}\right)dx,x,\sinh^{-1}(ax)\right)}{a^3}\\ &= -\frac{x^2\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}-\frac{\operatorname{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{4a^3}+\frac{3\operatorname{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx,x,\sinh^{-1}(a\right.}{4a^3}\\ &= -\frac{x^2\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}-\frac{\operatorname{Shi}\left(\sinh^{-1}(ax)\right)}{4a^3}+\frac{3\operatorname{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^3}\end{aligned}$$

Mathematica [A] time = 0.17, size = 49, normalized size = 0.91

$$-\frac{\frac{4a^2x^2\sqrt{a^2x^2+1}}{\sinh^{-1}(ax)}+\operatorname{Shi}\left(\sinh^{-1}(ax)\right)-3\operatorname{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a*x]^2,x]
```

```
[Out] -1/4*((4*a^2*x^2*Sqrt[1 + a^2*x^2])/ArcSinh[a*x] + SinhIntegral[ArcSinh[a*x
]] - 3*SinhIntegral[3*ArcSinh[a*x]])/a^3
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^2, x)
```

maple [A] time = 0.14, size = 56, normalized size = 1.04

$$\frac{\frac{\sqrt{a^2x^2+1}}{4\operatorname{arcsinh}(ax)} - \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{4} - \frac{\cosh(3\operatorname{arcsinh}(ax))}{4\operatorname{arcsinh}(ax)} + \frac{3\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/4/arcsinh(a*x)*(a^2*x^2+1)^(1/2)-1/4*Shi(arcsinh(a*x))-1/4/arcsinh
(a*x)*cosh(3*arcsinh(a*x))+3/4*Shi(3*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^5 + ax^3 + \left(a^2x^4 + x^2\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}\,a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{3\,a^5x^6 + 6\,a^3x^4 + 3\,ax^2 + \left(3\,a^3x^4 + ax^2\right)\left(a^2x^2+1\right) + \left(6\right.}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^5 + a*x^3 + (a^2*x^4 + x^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((3*a^5*x^6 +
 6*a^3*x^4 + 3*a*x^2 + (3*a^3*x^4 + a*x^2)*(a^2*x^2 + 1) + (6*a^4*x^5 + 7*a
^2*x^3 + 2*x)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2*a^3*
x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{asinh}(a\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a*x)^2,x)
```

```
[Out] int(x^2/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**2,x)
```

```
[Out] Integral(x**2/asinh(a*x)**2, x)
```

3.56 $\int \frac{x}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=37

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] Chi(2*arcsinh(a*x))/a^2-x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5665, 3301}

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^2,x]
```

```
[Out] -((x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + CoshIntegral[2*ArcSinh[a*x]]/a^
2
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sinh^{-1}(ax)^2}\,dx &= -\frac{x\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\mathrm{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^2}\\ &= -\frac{x\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)}+\frac{\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 0.86

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{a^2}-\frac{\sinh\left(2\sinh^{-1}(ax)\right)}{2a^2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a*x]^2,x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/a^2 - Sinh[2*ArcSinh[a*x]]/(2*a^2*ArcSinh[a*x]
)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(x/arcsinh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate(x/arcsinh(a*x)^2, x)
```

maple [A] time = 0.16, size = 28, normalized size = 0.76

$$\frac{-\frac{\sinh(2\operatorname{arcsinh}(ax))}{2\operatorname{arcsinh}(ax)} + \operatorname{X}(2\operatorname{arcsinh}(ax))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^2,x)
[Out] 1/a^2*(-1/2/arcsinh(a*x)*sinh(2*arcsinh(a*x))+Chi(2*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^4 + ax^2 + \left(a^2x^3 + x\right)\sqrt{a^2x^2+1}}{\left(a^3x^2 + \sqrt{a^2x^2+1}\,a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{2\,a^5x^5 + 2\left(a^2x^2+1\right)a^3x^3 + 4\,a^3x^3 + 2\,ax + \left(4\,a\right.}{\left(a^5x^4 + \left(a^2x^2+1\right)a^3x^2 + 2\,a^3x^2 + 2\left(a^4x^3 + a^2x\right)\sqrt{a^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="maxima")
[Out] -(a^3*x^4 + a*x^2 + (a^2*x^3 + x)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x
^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((2*a^5*x^5 + 2
*(a^2*x^2 + 1)*a^3*x^3 + 4*a^3*x^3 + 2*a*x + (4*a^4*x^4 + 4*a^2*x^2 + 1)*sq
rt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2*a^3*x^2 + 2*(a^4*x^3
 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\operatorname{asinh}(a\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a*x)^2,x)
[Out] int(x/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**2,x)
```

```
[Out] Integral(x/asinh(a*x)**2, x)
```

3.57 $\int \frac{1}{\sinh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=34

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{a} - \frac{\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

```
[Out] Shi(arcsinh(a*x))/a-(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] time = 0.08, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5655, 5779, 3298}

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{a} - \frac{\sqrt{a^2x^2+1}}{a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/a
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sinh^{-1}(ax)^2} \, dx &= -\frac{\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + a\int \frac{x}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)} \, dx \\
&= -\frac{\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x} \, dx, x, \sinh^{-1}(ax)\right)}{a} \\
&= -\frac{\sqrt{1+a^2x^2}}{a\sinh^{-1}(ax)} + \frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 0.91

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right) - \frac{\sqrt{a^2x^2+1}}{\sinh^{-1}(ax)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(-2),x]
```

```
[Out] (-(Sqrt[1 + a^2*x^2]/ArcSinh[a*x]) + SinhIntegral[ArcSinh[a*x]])/a
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^(-2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-2), x)
```

maple [A] time = 0.09, size = 30, normalized size = 0.88

$$\frac{-\frac{\sqrt{a^2x^2+1}}{\operatorname{arcsinh}(ax)} + \operatorname{Shi}(\operatorname{arcsinh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^2,x)
```

```
[Out] 1/a*(-1/arcsinh(a*x)*(a^2*x^2+1)^(1/2)+Shi(arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^3 + ax + \left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^3x^2 + \sqrt{a^2x^2+1}\,a^2x + a\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} + \int \frac{a^4x^4 + 2\,a^2x^2 + \left(a^2x^2+1\right)\left(a^2x^2-1\right) + \left(2\,a^3x^3\right.}{\left(a^4x^4 + \left(a^2x^2+1\right)a^2x^2 + 2\,a^2x^2 + 2\left(a^3x^3+ax\right)\sqrt{a^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x
+ a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((a^4*x^4 + 2*a^2*x^2 + (a^2*
x^2 + 1)*(a^2*x^2 - 1) + (2*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^4*x^4
 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1)
+ 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\operatorname{asinh}(a\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a*x)^2,x)
```

```
[Out] int(1/asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**2,x)
```

```
[Out] Integral(asinh(a*x)**(-2), x)
```

3.58 $\int \frac{1}{x\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x\sinh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^2),x]
[Out] Defer[Int][1/(x*ArcSinh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x\sinh^{-1}(ax)^2}\,dx=\int \frac{1}{x\sinh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^2),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^2), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\,\text{arsinh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\text{arsinh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^2), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^2,x)
```

```
[Out] int(1/x/arcsinh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^3 + ax + \left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^3x^3 + \sqrt{a^2x^2+1}\,a^2x^2 + ax\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} - \int \frac{2\left(a^2x^2+1\right)ax + \left(2\,a^2x^2 +\right.}{\left(a^5x^6 + \left(a^2x^2+1\right)a^3x^4 + 2\,a^3x^4 + ax^2 + 2\left(a^4x^5 +\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^3 + sqrt(a^2*x^2 + 1)*a^2*x^
2 + a*x)*log(a*x + sqrt(a^2*x^2 + 1))) - integrate((2*(a^2*x^2 + 1)*a*x + (
2*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^5*x^6 + (a^2*x^2 + 1)*a^3*x^4 + 2*a^3
*x^4 + a*x^2 + 2*(a^4*x^5 + a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*
x^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x\,\mathrm{asinh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)^2),x)
```

```
[Out] int(1/(x*asinh(a*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^{2}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(x*asinh(a*x)**2), x)
```

3.59 $\int \frac{1}{x^2 \sinh^{-1}(ax)^2}\, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \sinh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^2}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcSinh[a*x]^2),x]
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^2}\, dx = \int \frac{1}{x^2 \sinh^{-1}(ax)^2}\, dx$$

Mathematica [A] time = 5.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^2}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcSinh[a*x]^2),x]
[Out] Integrate[1/(x^2*ArcSinh[a*x]^2), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \operatorname{arsinh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/(x^2*arcsinh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arsinh}(ax)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate(1/(x^2*arcsinh(a*x)^2), x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)^2,x)
```

```
[Out] int(1/x^2/arcsinh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3x^3 + ax + \left(a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^3x^4 + \sqrt{a^2x^2+1}\,a^2x^3 + ax^2\right)\log\left(ax + \sqrt{a^2x^2+1}\right)} - \int \frac{a^5x^5 + 2\,a^3x^3 + \left(a^3x^3 + 3\,ax\right)\left(a^2x^2+1\right) + ax}{\left(a^5x^7 + \left(a^2x^2+1\right)a^3x^5 + 2\,a^3x^5 + ax^3 + 2\left(a^4x^6 +\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^4 + sqrt(a^2*x^2 + 1)*a^2*x^
3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^5*x^5 + 2*a^3*x^3 +
 (a^3*x^3 + 3*a*x)*(a^2*x^2 + 1) + a*x + (2*a^4*x^4 + 5*a^2*x^2 + 2)*sqrt(a
^2*x^2 + 1))/((a^5*x^7 + (a^2*x^2 + 1)*a^3*x^5 + 2*a^3*x^5 + a*x^3 + 2*(a^4
*x^6 + a^2*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{asinh}(a\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*asinh(a*x)^2),x)
```

```
[Out] int(1/(x^2*asinh(a*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(x**2*asinh(a*x)**2), x)
```

3.60 $\int \frac{x^4}{\sinh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=97

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{16a^5}-\frac{27\text{Chi}\left(3\sinh^{-1}(ax)\right)}{32a^5}+\frac{25\text{Chi}\left(5\sinh^{-1}(ax)\right)}{32a^5}-\frac{2x^3}{a^2\sinh^{-1}(ax)}-\frac{x^4\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{5x^5}{2\sinh^{-1}(ax)}$$

```
[Out] -2*x^3/a^2/arcsinh(a*x)-5/2*x^5/arcsinh(a*x)+1/16*Chi(arcsinh(a*x))/a^5-27/
32*Chi(3*arcsinh(a*x))/a^5+25/32*Chi(5*arcsinh(a*x))/a^5-1/2*x^4*(a^2*x^2+1
)^(1/2)/a/arcsinh(a*x)^2
```

Rubi [A] time = 0.35, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5667, 5774, 5669, 5448, 3301}

$$\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{16a^5}-\frac{27\text{Chi}\left(3\sinh^{-1}(ax)\right)}{32a^5}+\frac{25\text{Chi}\left(5\sinh^{-1}(ax)\right)}{32a^5}-\frac{x^4\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{2x^3}{a^2\sinh^{-1}(ax)}-\frac{5x^5}{2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^3,x]
```

```
[Out] -(x^4*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - (2*x^3)/(a^2*ArcSinh[a*x])
- (5*x^5)/(2*ArcSinh[a*x]) + CoshIntegral[ArcSinh[a*x]]/(16*a^5) - (27*Cosh
Integral[3*ArcSinh[a*x]])/(32*a^5) + (25*CoshIntegral[5*ArcSinh[a*x]])/(32*
a^5)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^3}\,dx = -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} + \frac{2\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx}{a} + \frac{1}{2}(5a)\int \frac{x^5}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{25}{2}\int \frac{x^4}{\sinh^{-1}(ax)}\,dx + \frac{6\int \frac{x^2}{\sinh^{-1}(ax)}}{a^2}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{6\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \sinh^{-}\right.}{a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{6\,\mathrm{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right) dx, x\right.}{a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{25\,\mathrm{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \sinh^{-1}(ax\right.}{32a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{2x^3}{a^2\sinh^{-1}(ax)} - \frac{5x^5}{2\sinh^{-1}(ax)} + \frac{\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{16a^5} - \frac{27\mathrm{Chi}\left(3\sinh^{-1}\right.}{32a^5}$$

Mathematica [A] time = 0.16, size = 102, normalized size = 1.05

$$-\frac{80a^5x^5\sinh^{-1}(ax) + 64a^3x^3\sinh^{-1}(ax) + 16a^4x^4\sqrt{a^2x^2+1} - 2\sinh^{-1}(ax)^2\mathrm{Chi}\left(\sinh^{-1}(ax)\right) + 27\sinh^{-1}(ax}{32a^5\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcSinh[a*x]^3,x]
```

```
[Out] -1/32*(16*a^4*x^4*Sqrt[1 + a^2*x^2] + 64*a^3*x^3*ArcSinh[a*x] + 80*a^5*x^5*
ArcSinh[a*x] - 2*ArcSinh[a*x]^2*CoshIntegral[ArcSinh[a*x]] + 27*ArcSinh[a*x
]^2*CoshIntegral[3*ArcSinh[a*x]] - 25*ArcSinh[a*x]^2*CoshIntegral[5*ArcSinh
[a*x]])/(a^5*ArcSinh[a*x]^2)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^4}{\mathrm{arsinh}\,(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\mathrm{arsinh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^3, x)
```

maple [A] time = 0.17, size = 120, normalized size = 1.24

$$\frac{-\frac{\sqrt{a^2x^2+1}}{16\operatorname{arcsinh}(ax)^2}-\frac{ax}{16\operatorname{arcsinh}(ax)}+\frac{\mathrm{X}(\operatorname{arcsinh}(ax))}{16}+\frac{3\cosh(3\operatorname{arcsinh}(ax))}{32\operatorname{arcsinh}(ax)^2}+\frac{9\sinh(3\operatorname{arcsinh}(ax))}{32\operatorname{arcsinh}(ax)}-\frac{27\mathrm{X}(3\operatorname{arcsinh}(ax))}{32}-\frac{\cosh(5\operatorname{arcsinh}}{32\operatorname{arcsinh}(a}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^3,x)
```

```
[Out] 1/a^5*(-1/16/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1/16*a*x/arcsinh(a*x)+1/16*Ch
i(arcsinh(a*x))+3/32/arcsinh(a*x)^2*cosh(3*arcsinh(a*x))+9/32/arcsinh(a*x)*
sinh(3*arcsinh(a*x))-27/32*Chi(3*arcsinh(a*x))-1/32/arcsinh(a*x)^2*cosh(5*a
rcsinh(a*x))-5/32/arcsinh(a*x)*sinh(5*arcsinh(a*x))+25/32*Chi(5*arcsinh(a*x
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^8x^{11}+3a^6x^9+3a^4x^7+a^2x^5+\left(a^5x^8+a^3x^6\right)\left(a^2x^2+1\right)^{\frac{3}{2}}+\left(3a^6x^9+5a^4x^7+2a^2x^5\right)\left(a^2x^2+1\right)+\left(5a^8x^{11}+\right.}{2\left(a^8x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^11 + 3*a^6*x^9 + 3*a^4*x^7 + a^2*x^5 + (a^5*x^8 + a^3*x^6)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^9 + 5*a^4*x^7 + 2*a^2*x^5)*(a^2*x^2 + 1) + (5*a^
8*x^11 + 15*a^6*x^9 + 15*a^4*x^7 + 5*a^2*x^5 + (5*a^5*x^8 + 8*a^3*x^6 + 3*a
*x^4)*(a^2*x^2 + 1)^(3/2) + (15*a^6*x^9 + 31*a^4*x^7 + 20*a^2*x^5 + 4*x^3)*
(a^2*x^2 + 1) + (15*a^7*x^10 + 38*a^5*x^8 + 32*a^3*x^6 + 9*a*x^4)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^10 + 7*a^5*x^8 + 5*a^3*x^
6 + a*x^4)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a
^5*x^3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5
 + 2*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) +
integrate(1/2*(25*a^10*x^12 + 100*a^8*x^10 + 150*a^6*x^8 + 100*a^4*x^6 + 25
*a^2*x^4 + (25*a^6*x^8 + 24*a^4*x^6 + 3*a^2*x^4)*(a^2*x^2 + 1)^2 + (100*a^7
*x^9 + 172*a^5*x^7 + 87*a^3*x^5 + 12*a*x^3)*(a^2*x^2 + 1)^(3/2) + 3*(50*a^8
*x^10 + 124*a^6*x^8 + 105*a^4*x^6 + 35*a^2*x^4 + 4*x^2)*(a^2*x^2 + 1) + (10
0*a^9*x^11 + 324*a^7*x^9 + 381*a^5*x^7 + 193*a^3*x^5 + 36*a*x^3)*sqrt(a^2*x
^2 + 1))/((a^10*x^8 + 4*a^8*x^6 + (a^2*x^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a
^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4
 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 4*(a^9*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*
x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{asinh}(a\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asinh(a*x)^3,x)
```

```
[Out] int(x^4/asinh(a*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**3,x)
```

```
[Out] Integral(x**4/asinh(a*x)**3, x)
```

3.61 $\int \frac{x^3}{\sinh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=82

$$-\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{a^4}-\frac{3x^2}{2a^2\sinh^{-1}(ax)}-\frac{x^3\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{2x^4}{\sinh^{-1}(ax)}$$

```
[Out] -3/2*x^2/a^2/arcsinh(a*x)-2*x^4/arcsinh(a*x)-1/2*Shi(2*arcsinh(a*x))/a^4+Sh
i(4*arcsinh(a*x))/a^4-1/2*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2
```

Rubi [A] time = 0.30, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5667, 5774, 5669, 5448, 3298, 12}

$$-\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^4}+\frac{\text{Shi}\left(4\sinh^{-1}(ax)\right)}{a^4}-\frac{x^3\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{3x^2}{2a^2\sinh^{-1}(ax)}-\frac{2x^4}{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^3,x]
```

```
[Out] -(x^3*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - (3*x^2)/(2*a^2*ArcSinh[a*x]
) - (2*x^4)/ArcSinh[a*x] - SinhIntegral[2*ArcSinh[a*x]]/(2*a^4) + SinhInteg
ral[4*ArcSinh[a*x]]/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^3}\,dx = -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} + \frac{3\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx}{2a} + (2a)\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} + 8\int \frac{x^3}{\sinh^{-1}(ax)}\,dx + \frac{3\int \frac{x}{\sinh^{-1}(ax)}\,dx}{a^2}$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(\ldots\right)}{a^4}$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^4} + \ldots$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{3x^2}{2a^2\sinh^{-1}(ax)} - \frac{2x^4}{\sinh^{-1}(ax)} - \frac{\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{2a^4} + \frac{\mathrm{Shi}\left(4\sinh^{-1}(a\ldots\right)}{a^4}$$

Mathematica [A] time = 0.22, size = 69, normalized size = 0.84

$$-\frac{\frac{a^2x^2\left(ax\sqrt{a^2x^2+1}+\left(4a^2x^2+3\right)\sinh^{-1}(ax)\right)}{\sinh^{-1}(ax)^2} + \mathrm{Shi}\left(2\sinh^{-1}(ax)\right) - 2\mathrm{Shi}\left(4\sinh^{-1}(ax)\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcSinh[a*x]^3,x]
```

```
[Out] -1/2*((a^2*x^2*(a*x*Sqrt[1 + a^2*x^2] + (3 + 4*a^2*x^2)*ArcSinh[a*x]))/ArcS
inh[a*x]^2 + SinhIntegral[2*ArcSinh[a*x]] - 2*SinhIntegral[4*ArcSinh[a*x]])
/a^4
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^3}{\operatorname{arsinh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.16, size = 82, normalized size = 1.00

$$\frac{\frac{\sinh(2\,\mathrm{arcsinh}(ax))}{8\,\mathrm{arcsinh}(ax)^2}+\frac{\cosh(2\,\mathrm{arcsinh}(ax))}{4\,\mathrm{arcsinh}(ax)}-\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(ax))}{2}-\frac{\sinh(4\,\mathrm{arcsinh}(ax))}{16\,\mathrm{arcsinh}(ax)^2}-\frac{\cosh(4\,\mathrm{arcsinh}(ax))}{4\,\mathrm{arcsinh}(ax)}+\mathrm{Shi}\left(4\,\mathrm{arcsinh}\left(ax\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^3,x)
```

```
[Out] 1/a^4*(1/8/arcsinh(a*x)^2*sinh(2*arcsinh(a*x))+1/4/arcsinh(a*x)*cosh(2*arcs
inh(a*x))-1/2*Shi(2*arcsinh(a*x))-1/16/arcsinh(a*x)^2*sinh(4*arcsinh(a*x))-
1/4/arcsinh(a*x)*cosh(4*arcsinh(a*x))+Shi(4*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^8x^{10}+3\,a^6x^8+3\,a^4x^6+a^2x^4+\left(a^5x^7+a^3x^5\right)\left(a^2x^2+1\right)^{\frac{3}{2}}+\left(3\,a^6x^8+5\,a^4x^6+2\,a^2x^4\right)\left(a^2x^2+1\right)+\left(4\,a^8x^{10}+\right.}{2\left(a^8x^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^10 + 3*a^6*x^8 + 3*a^4*x^6 + a^2*x^4 + (a^5*x^7 + a^3*x^5)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^8 + 5*a^4*x^6 + 2*a^2*x^4)*(a^2*x^2 + 1) + (4*a^
8*x^10 + 12*a^6*x^8 + 12*a^4*x^6 + 4*a^2*x^4 + 2*(2*a^5*x^7 + 3*a^3*x^5 + a
*x^3)*(a^2*x^2 + 1)^(3/2) + 3*(4*a^6*x^8 + 8*a^4*x^6 + 5*a^2*x^4 + x^2)*(a^
2*x^2 + 1) + (12*a^7*x^9 + 30*a^5*x^7 + 25*a^3*x^5 + 7*a*x^3)*sqrt(a^2*x^2
+ 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^9 + 7*a^5*x^7 + 5*a^3*x^5 + a
*x^3)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^
3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*
a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integ
rate(1/2*(16*a^10*x^11 + 64*a^8*x^9 + 96*a^6*x^7 + 64*a^4*x^5 + 16*a^2*x^3
+ 4*(4*a^6*x^7 + 3*a^4*x^5)*(a^2*x^2 + 1)^2 + (64*a^7*x^8 + 100*a^5*x^6 + 4
2*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1)^(3/2) + 6*(16*a^8*x^9 + 38*a^6*x^7 + 30*
a^4*x^5 + 9*a^2*x^3 + x)*(a^2*x^2 + 1) + (64*a^9*x^10 + 204*a^7*x^8 + 234*a
^5*x^6 + 115*a^3*x^4 + 21*a*x^2)*sqrt(a^2*x^2 + 1))/((a^10*x^8 + 4*a^8*x^6
+ (a^2*x^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(
a^2*x^2 + 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2
+ 4*(a^9*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x +
sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\mathrm{asinh}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asinh(a*x)^3,x)
```

```
[Out] int(x^3/asinh(a*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**3,x)
```

```
[Out] Integral(x**3/asinh(a*x)**3, x)
```

3.62 $\int \frac{x^2}{\sinh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=81

$$-\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(ax)\right)}{8a^3}-\frac{x^2\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{x}{a^2\sinh^{-1}(ax)}-\frac{3x^3}{2\sinh^{-1}(ax)}$$

```
[Out] -x/a^2/arcsinh(a*x)-3/2*x^3/arcsinh(a*x)-1/8*Chi(arcsinh(a*x))/a^3+9/8*Chi(
3*arcsinh(a*x))/a^3-1/2*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2
```

Rubi [A] time = 0.25, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5667, 5774, 5669, 5448, 3301, 5657}

$$-\frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(ax)\right)}{8a^3}-\frac{x^2\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{x}{a^2\sinh^{-1}(ax)}-\frac{3x^3}{2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^3,x]
```

```
[Out] -(x^2*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - x/(a^2*ArcSinh[a*x]) - (3*x
^3)/(2*ArcSinh[a*x]) - CoshIntegral[ArcSinh[a*x]]/(8*a^3) + (9*CoshIntegral
[3*ArcSinh[a*x]])/(8*a^3)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^3}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} + \frac{\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx}{a} + \frac{1}{2}(3a)\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} + \frac{9}{2}\int \frac{x^2}{\sinh^{-1}(ax)}\,dx + \frac{\int \frac{1}{\sinh^{-1}(ax)}\,dx}{a^2}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^3} +$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} + \frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{a^3} + \frac{9\,\text{Subst}\left(\int \left(-\frac{\text{co}}{}\right.\right.}{}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} + \frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{a^3} - \frac{9\,\text{Subst}\left(\int \frac{\cosh(}{x}\right.}{}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2} - \frac{x}{a^2\sinh^{-1}(ax)} - \frac{3x^3}{2\sinh^{-1}(ax)} - \frac{\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a^3} + \frac{9\text{Chi}\left(3\sinh^{-1}(\right.}{8a^3}$$

Mathematica [A] time = 0.16, size = 64, normalized size = 0.79

$$-\frac{\frac{4ax\left(ax\sqrt{a^2x^2+1}+\left(3a^2x^2+2\right)\sinh^{-1}(ax)\right)}{\sinh^{-1}(ax)^2}+\text{Chi}\left(\sinh^{-1}(ax)\right)-9\text{Chi}\left(3\sinh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a*x]^3,x]
```

```
[Out] -1/8*((4*a*x*(a*x*Sqrt[1 + a^2*x^2] + (2 + 3*a^2*x^2)*ArcSinh[a*x]))/ArcSin
h[a*x]^2 + CoshIntegral[ArcSinh[a*x]] - 9*CoshIntegral[3*ArcSinh[a*x]])/a^3
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\text{arsinh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\text{arsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^3, x)
```

maple [A] time = 0.14, size = 81, normalized size = 1.00

$$\frac{\frac{\sqrt{a^2x^2+1}}{8\,\mathrm{arcsinh}(ax)^2}+\frac{ax}{8\,\mathrm{arcsinh}(ax)}-\frac{\mathrm{X}(\mathrm{arcsinh}(ax))}{8}-\frac{\cosh(3\,\mathrm{arcsinh}(ax))}{8\,\mathrm{arcsinh}(ax)^2}-\frac{3\sinh(3\,\mathrm{arcsinh}(ax))}{8\,\mathrm{arcsinh}(ax)}+\frac{9\mathrm{X}(3\,\mathrm{arcsinh}(ax))}{8}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/8/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+1/8*a*x/arcsinh(a*x)-1/8*Chi(ar
csinh(a*x))-1/8/arcsinh(a*x)^2*cosh(3*arcsinh(a*x))-3/8/arcsinh(a*x)*sinh(3
*arcsinh(a*x))+9/8*Chi(3*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^8x^9+3\,a^6x^7+3\,a^4x^5+a^2x^3+\left(a^5x^6+a^3x^4\right)\left(a^2x^2+1\right)^{\frac{3}{2}}+\left(3\,a^6x^7+5\,a^4x^5+2\,a^2x^3\right)\left(a^2x^2+1\right)+\left(3\,a^8x^9+9\right.}{2\left(a^8x^6+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^9 + 3*a^6*x^7 + 3*a^4*x^5 + a^2*x^3 + (a^5*x^6 + a^3*x^4)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^7 + 5*a^4*x^5 + 2*a^2*x^3)*(a^2*x^2 + 1) + (3*a^8
*x^9 + 9*a^6*x^7 + 9*a^4*x^5 + 3*a^2*x^3 + (3*a^5*x^6 + 4*a^3*x^4 + a*x^2)*
(a^2*x^2 + 1)^(3/2) + (9*a^6*x^7 + 17*a^4*x^5 + 10*a^2*x^3 + 2*x)*(a^2*x^2
+ 1) + (9*a^7*x^8 + 22*a^5*x^6 + 18*a^3*x^4 + 5*a*x^2)*sqrt(a^2*x^2 + 1))*l
og(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^8 + 7*a^5*x^6 + 5*a^3*x^4 + a*x^2)*s
qrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^3 + 3*a
^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*a^5*x^3
 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/
2*(9*a^10*x^10 + 36*a^8*x^8 + 54*a^6*x^6 + 36*a^4*x^4 + 9*a^2*x^2 + (9*a^6*
x^6 + 4*a^4*x^4 - a^2*x^2)*(a^2*x^2 + 1)^2 + (36*a^7*x^7 + 48*a^5*x^5 + 13*
a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^(3/2) + (54*a^8*x^8 + 120*a^6*x^6 + 83*a^4*x
^4 + 19*a^2*x^2 + 2)*(a^2*x^2 + 1) + (36*a^9*x^9 + 112*a^7*x^7 + 123*a^5*x^
5 + 57*a^3*x^3 + 10*a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^8 + 4*a^8*x^6 + (a^2*x
^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(a^2*x^2
+ 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 4*(a^9
*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2
*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\mathrm{asinh}\left(a\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a*x)^3,x)
```

```
[Out] int(x^2/asinh(a*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**3,x)
```

```
[Out] Integral(x**2/asinh(a*x)**3, x)
```

3.63 $\int \frac{x}{\sinh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=63

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}$$

```
[Out] -1/2/a^2/arcsinh(a*x)-x^2/arcsinh(a*x)+Shi(2*arcsinh(a*x))/a^2-1/2*x*(a^2*x
^2+1)^(1/2)/a/arcsinh(a*x)^2
```

Rubi [A] time = 0.17, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5667, 5774, 5669, 5448, 12, 3298, 5675}

$$\frac{\text{Shi}\left(2\sinh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^3,x]
```

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]^2) - 1/(2*a^2*ArcSinh[a*x]) - x^2/
ArcSinh[a*x] + SinhIntegral[2*ArcSinh[a*x]]/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sinh^{-1}(ax)^3}\,dx &= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}+\frac{\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx}{2a}+a\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}\,dx\\
&= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}+2\int \frac{x}{\sinh^{-1}(ax)}\,dx\\
&= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}+\frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx,x,\sinh^{-1}(\right.}{a^2}\\
&= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}+\frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx,x,\sinh^{-1}(ax)\right)}{a^2}\\
&= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}+\frac{\mathrm{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a^2}\\
&= -\frac{x\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{1}{2a^2\sinh^{-1}(ax)}-\frac{x^2}{\sinh^{-1}(ax)}+\frac{\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.98

$$\frac{\mathrm{Shi}\left(2\sinh^{-1}(ax)\right)}{a^2}-\frac{x\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}+\frac{-2a^2x^2-1}{2a^2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a*x]^3,x]
```

```
[Out] -1/2*(x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) + (-1 - 2*a^2*x^2)/(2*a^2*Arc
Sinh[a*x]) + SinhIntegral[2*ArcSinh[a*x]]/a^2
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{\mathrm{arsinh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(a*x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{arsinh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^3, x)
```

maple [A] time = 0.17, size = 43, normalized size = 0.68

$$\frac{-\frac{\sinh(2\,\mathrm{arcsinh}(ax))}{4\,\mathrm{arcsinh}(ax)^2}-\frac{\cosh(2\,\mathrm{arcsinh}(ax))}{2\,\mathrm{arcsinh}(ax)}+\mathrm{Shi}\left(2\,\mathrm{arcsinh}\left(ax\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^3,x)
```

```
[Out] 1/a^2*(-1/4/arcsinh(a*x)^2*sinh(2*arcsinh(a*x))-1/2/arcsinh(a*x)*cosh(2*arc
sinh(a*x))+Shi(2*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^8x^8+3\,a^6x^6+3\,a^4x^4+a^2x^2+\left(a^5x^5+a^3x^3\right)\left(a^2x^2+1\right)^{\frac{3}{2}}+\left(3\,a^6x^6+5\,a^4x^4+2\,a^2x^2\right)\left(a^2x^2+1\right)+\left(2\,a^8x^8+6\right.}{2\left(a^8x^6+3\,a^6x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) + (2*a^8
*x^8 + 6*a^6*x^6 + 6*a^4*x^4 + 2*a^2*x^2 + 2*(a^5*x^5 + a^3*x^3)*(a^2*x^2 +
 1)^(3/2) + (6*a^6*x^6 + 10*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1) + (6*a^7
*x^7 + 14*a^5*x^5 + 11*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a
^2*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))
/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^3 + 3*a^4*x^2 + 3*(a^6*x
^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*a^5*x^3 + a^3*x)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(4*a^9*x^9 + 16
*a^7*x^7 + 4*(a^2*x^2 + 1)^2*a^5*x^5 + 24*a^5*x^5 + 16*a^3*x^3 + (16*a^6*x^
6 + 16*a^4*x^4 - 3)*(a^2*x^2 + 1)^(3/2) + 24*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3
)*(a^2*x^2 + 1) + 4*a*x + (16*a^8*x^8 + 48*a^6*x^6 + 48*a^4*x^4 + 19*a^2*x^
2 + 3)*sqrt(a^2*x^2 + 1))/((a^9*x^8 + 4*a^7*x^6 + (a^2*x^2 + 1)^2*a^5*x^4 +
 6*a^5*x^4 + 4*a^3*x^2 + 4*(a^6*x^5 + a^4*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^7
*x^6 + 2*a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1) + 4*(a^8*x^7 + 3*a^6*x^5 + 3*a^4*
x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{{\mathrm{asinh}\left(a\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a*x)^3,x)
```

```
[Out] int(x/asinh(a*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^{3}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**3,x)
```

```
[Out] Integral(x/asinh(a*x)**3, x)
```

3.64 $\int \frac{1}{\sinh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}+\frac{\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{2a}-\frac{x}{2\sinh^{-1}(ax)}$$

```
[Out] -1/2*x/arcsinh(a*x)+1/2*Chi(arcsinh(a*x))/a-1/2*(a^2*x^2+1)^(1/2)/a/arcsinh
(a*x)^2
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5655, 5774, 5657, 3301}

$$-\frac{\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)^2}+\frac{\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{2a}-\frac{x}{2\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-3),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(2*a*ArcSinh[a*x]^2) - x/(2*ArcSinh[a*x]) + CoshIntegral
[ArcSinh[a*x]]/(2*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{-1}(ax)^3}\,dx &= -\frac{\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}+\frac{1}{2}a\int \frac{x}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^2}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{x}{2\sinh^{-1}(ax)}+\frac{1}{2}\int \frac{1}{\sinh^{-1}(ax)}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{x}{2\sinh^{-1}(ax)}+\frac{\operatorname{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{2a}\\ &= -\frac{\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)^2}-\frac{x}{2\sinh^{-1}(ax)}+\frac{\operatorname{Chi}\left(\sinh^{-1}(ax)\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.94

$$-\frac{\sqrt{a^2x^2+1}+\sinh^{-1}(ax)^2\left(-\operatorname{Chi}\left(\sinh^{-1}(ax)\right)\right)+ax\sinh^{-1}(ax)}{2a\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(-3),x]
```

```
[Out] -1/2*(Sqrt[1 + a^2*x^2] + a*x*ArcSinh[a*x] - ArcSinh[a*x]^2*CoshIntegral[Ar
cSinh[a*x]])/(a*ArcSinh[a*x]^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{\operatorname{arsinh}(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^(-3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-3), x)
```

maple [A] time = 0.09, size = 42, normalized size = 0.84

$$\frac{-\frac{\sqrt{a^2x^2+1}}{2\operatorname{arcsinh}(ax)^2}-\frac{ax}{2\operatorname{arcsinh}(ax)}+\frac{\mathrm{X}(\operatorname{arcsinh}(ax))}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^3,x)
```

```
[Out] 1/a*(-1/2/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1/2*a*x/arcsinh(a*x)+1/2*Chi(arc
sinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^7x^7+3\,a^5x^5+3\,a^3x^3+\left(a^4x^4+a^2x^2\right)\left(a^2x^2+1\right)^{\frac{3}{2}}+\left(3\,a^5x^5+5\,a^3x^3+2\,ax\right)\left(a^2x^2+1\right)+ax+\left(a^7x^7+3\,a^5x^5\right.}{2\left(a^7x^6+3\,a^5x^4+\left(a^2x^2+1\right)^{\frac{3}{2}}a^4x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*x^7 + 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1)^(
3/2) + (3*a^5*x^5 + 5*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1) + a*x + (a^7*x^7 + 3*a
^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - 1)*(a^2*x^2 + 1)^(3/2) + 3*(a^5*x^5 + a^3*x
^3)*(a^2*x^2 + 1) + a*x + (3*a^6*x^6 + 6*a^4*x^4 + 4*a^2*x^2 + 1)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^6*x^6 + 7*a^4*x^4 + 5*a^2*x^2
 + 1)*sqrt(a^2*x^2 + 1))/((a^7*x^6 + 3*a^5*x^4 + (a^2*x^2 + 1)^(3/2)*a^4*x^
3 + 3*a^3*x^2 + 3*(a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1) + 3*(a^6*x^5 + 2*a^4*x^
3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))^2) + integra
te(1/2*(a^8*x^8 + 4*a^6*x^6 + 6*a^4*x^4 + 4*a^2*x^2 + (a^4*x^4 + 3)*(a^2*x^
2 + 1)^2 + (4*a^5*x^5 + 4*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1)^(3/2) + 3*(2*a^6*x
^6 + 4*a^4*x^4 + a^2*x^2 - 1)*(a^2*x^2 + 1) + (4*a^7*x^7 + 12*a^5*x^5 + 9*a
^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^8*x^8 + 4*a^6*x^6 + (a^2*x^2 + 1)^
2*a^4*x^4 + 6*a^4*x^4 + 4*a^2*x^2 + 4*(a^5*x^5 + a^3*x^3)*(a^2*x^2 + 1)^(3/
2) + 6*(a^6*x^6 + 2*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1) + 4*(a^7*x^7 + 3*a^5*x
^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{asinh}\left(a\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a*x)^3,x)
```

```
[Out] int(1/asinh(a*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^{3}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**3,x)
```

```
[Out] Integral(asinh(a*x)**(-3), x)
```

3.65 $\int \frac{1}{x\sinh^{-1}(ax)^3} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x\sinh^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^3,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sinh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^3),x]
[Out] Defer[Int][1/(x*ArcSinh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x\sinh^{-1}(ax)^3} dx = \int \frac{1}{x\sinh^{-1}(ax)^3} dx$$

Mathematica [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sinh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^3),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^3), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\,\text{arsinh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(a*x)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\text{arsinh}(ax)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^3), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^3,x)
```

```
[Out] int(1/x/arcsinh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^8x^8 + 3\,a^6x^6 + 3\,a^4x^4 + a^2x^2 + \left(a^5x^5 + a^3x^3\right)\left(a^2x^2+1\right)^{\frac{3}{2}} + \left(3\,a^6x^6 + 5\,a^4x^4 + 2\,a^2x^2\right)\left(a^2x^2+1\right) - \left(2\left(a^3x^3 + a\right.\right.}{2\left(a^8x^8 + 3\,a^6x^6 + \left(a^2x^2+1\right)^{\frac{3}{2}}a^5x^5 + 3\,a^4x^4 + a^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) - (2*(a^
3*x^3 + a*x)*(a^2*x^2 + 1)^(3/2) + (4*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1
) + (2*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^8
*x^8 + 3*a^6*x^6 + (a^2*x^2 + 1)^(3/2)*a^5*x^5 + 3*a^4*x^4 + a^2*x^2 + 3*(a
^6*x^6 + a^4*x^4)*(a^2*x^2 + 1) + 3*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(4*(a^4*x^4 + 2
*a^2*x^2)*(a^2*x^2 + 1)^2 + (12*a^5*x^5 + 22*a^3*x^3 + 7*a*x)*(a^2*x^2 + 1)
^(3/2) + 2*(6*a^6*x^6 + 10*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1) + (4*a^7*
x^7 + 6*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^11 + 4*a^8*x
^9 + (a^2*x^2 + 1)^2*a^6*x^7 + 6*a^6*x^7 + 4*a^4*x^5 + a^2*x^3 + 4*(a^7*x^8
 + a^5*x^6)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^9 + 2*a^6*x^7 + a^4*x^5)*(a^2*x^
2 + 1) + 4*(a^9*x^10 + 3*a^7*x^8 + 3*a^5*x^6 + a^3*x^4)*sqrt(a^2*x^2 + 1))*
log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{asinh}(a\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)^3),x)
```

```
[Out] int(1/(x*asinh(a*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**3,x)
```

```
[Out] Integral(1/(x*asinh(a*x)**3), x)
```

3.66 $\int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \sinh^{-1}(ax)^3}, x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)^3,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcSinh[a*x]^3),x]
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx = \int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 5.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcSinh[a*x]^3),x]
[Out] Integrate[1/(x^2*ArcSinh[a*x]^3), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \operatorname{arsinh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="fricas")
[Out] integral(1/(x^2*arcsinh(a*x)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arsinh}(ax)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="giac")
[Out] integrate(1/(x^2*arcsinh(a*x)^3), x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)^3,x)
```

```
[Out] int(1/x^2/arcsinh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^8x^8 + 3\,a^6x^6 + 3\,a^4x^4 + a^2x^2 + \left(a^5x^5 + a^3x^3\right)\left(a^2x^2+1\right)^{\frac{3}{2}} + \left(3\,a^6x^6 + 5\,a^4x^4 + 2\,a^2x^2\right)\left(a^2x^2+1\right) - \left(a^8x^8 + 3\,a^6\right.}{2\left(a^8x^9 + 3\,a^6x^7 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) - (a^8*x
^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + 4*a^3*x^3 + 3*a*x)*(a^2*x
^2 + 1)^(3/2) + (3*a^6*x^6 + 11*a^4*x^4 + 10*a^2*x^2 + 2)*(a^2*x^2 + 1) + (
3*a^7*x^7 + 10*a^5*x^5 + 10*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + s
qrt(a^2*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2
+ 1))/((a^8*x^9 + 3*a^6*x^7 + (a^2*x^2 + 1)^(3/2)*a^5*x^6 + 3*a^4*x^5 + a^2
*x^3 + 3*(a^6*x^7 + a^4*x^5)*(a^2*x^2 + 1) + 3*(a^7*x^8 + 2*a^5*x^6 + a^3*x
^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(a^1
0*x^10 + 4*a^8*x^8 + 6*a^6*x^6 + 4*a^4*x^4 + a^2*x^2 + (a^6*x^6 + 12*a^4*x^
4 + 15*a^2*x^2)*(a^2*x^2 + 1)^2 + (4*a^7*x^7 + 40*a^5*x^5 + 57*a^3*x^3 + 18
*a*x)*(a^2*x^2 + 1)^(3/2) + 3*(2*a^8*x^8 + 16*a^6*x^6 + 25*a^4*x^4 + 13*a^2
*x^2 + 2)*(a^2*x^2 + 1) + (4*a^9*x^9 + 24*a^7*x^7 + 39*a^5*x^5 + 25*a^3*x^3
 + 6*a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^12 + 4*a^8*x^10 + (a^2*x^2 + 1)^2*a^6
*x^8 + 6*a^6*x^8 + 4*a^4*x^6 + a^2*x^4 + 4*(a^7*x^9 + a^5*x^7)*(a^2*x^2 + 1
)^(3/2) + 6*(a^8*x^10 + 2*a^6*x^8 + a^4*x^6)*(a^2*x^2 + 1) + 4*(a^9*x^11 +
3*a^7*x^9 + 3*a^5*x^7 + a^3*x^5)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
+ 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{asinh}(a\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*asinh(a*x)^3),x)
```

```
[Out] int(1/(x^2*asinh(a*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{asinh}^3(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)**3,x)
```

```
[Out] Integral(1/(x**2*asinh(a*x)**3), x)
```

3.67 $\int \frac{x^4}{\sinh^{-1}(ax)^4} dx$

Optimal. Leaf size=155

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{48a^5}-\frac{27\text{Shi}\left(3\sinh^{-1}(ax)\right)}{32a^5}+\frac{125\text{Shi}\left(5\sinh^{-1}(ax)\right)}{96a^5}-\frac{2x^3}{3a^2\sinh^{-1}(ax)^2}-\frac{25x^4\sqrt{a^2x^2+1}}{6a\sinh^{-1}(ax)}-\frac{x^4\sqrt{a^2x}}{3a\sinh^{-}}$$

```
[Out] -2/3*x^3/a^2/arcsinh(a*x)^2-5/6*x^5/arcsinh(a*x)^2+1/48*Shi(arcsinh(a*x))/a
^5-27/32*Shi(3*arcsinh(a*x))/a^5+125/96*Shi(5*arcsinh(a*x))/a^5-1/3*x^4*(a^
2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-2*x^2*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-25
/6*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] time = 0.32, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5667, 5774, 5665, 3298}

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{48a^5}-\frac{27\text{Shi}\left(3\sinh^{-1}(ax)\right)}{32a^5}+\frac{125\text{Shi}\left(5\sinh^{-1}(ax)\right)}{96a^5}-\frac{25x^4\sqrt{a^2x^2+1}}{6a\sinh^{-1}(ax)}-\frac{x^4\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{2x^3}{3a^2\sinh^{-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^4,x]
```

```
[Out] -(x^4*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - (2*x^3)/(3*a^2*ArcSinh[a*x]
^2) - (5*x^5)/(6*ArcSinh[a*x]^2) - (2*x^2*Sqrt[1 + a^2*x^2])/(a^3*ArcSinh[a
*x]) - (25*x^4*Sqrt[1 + a^2*x^2])/(6*a*ArcSinh[a*x]) + SinhIntegral[ArcSinh
[a*x]]/(48*a^5) - (27*SinhIntegral[3*ArcSinh[a*x]])/(32*a^5) + (125*SinhInt
egral[5*ArcSinh[a*x]])/(96*a^5)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
```

```
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^4}\,dx = -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} + \frac{4\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx}{3a} + \frac{1}{3}(5a)\int \frac{x^5}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{2x^3}{3a^2\sinh^{-1}(ax)^2} - \frac{5x^5}{6\sinh^{-1}(ax)^2} + \frac{25}{6}\int \frac{x^4}{\sinh^{-1}(ax)^2}\,dx + \frac{2\int \frac{x^2}{\sinh^{-1}(ax}}{a^2}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{2x^3}{3a^2\sinh^{-1}(ax)^2} - \frac{5x^5}{6\sinh^{-1}(ax)^2} - \frac{2x^2\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{25x^4\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)} + $$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{2x^3}{3a^2\sinh^{-1}(ax)^2} - \frac{5x^5}{6\sinh^{-1}(ax)^2} - \frac{2x^2\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{25x^4\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)} - $$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{2x^3}{3a^2\sinh^{-1}(ax)^2} - \frac{5x^5}{6\sinh^{-1}(ax)^2} - \frac{2x^2\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{25x^4\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)} + $$

Mathematica [A] time = 0.37, size = 156, normalized size = 1.01

$$-\frac{80a^5x^5\sinh^{-1}(ax) + 64a^3x^3\sinh^{-1}(ax) + 192a^2x^2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2 + 32a^4x^4\sqrt{a^2x^2+1} + 400a^4x^4\sqrt{a^2x^2}}{96a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcSinh[a*x]^4,x]
```

```
[Out] -1/96*(32*a^4*x^4*Sqrt[1 + a^2*x^2] + 64*a^3*x^3*ArcSinh[a*x] + 80*a^5*x^5*
ArcSinh[a*x] + 192*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 + 400*a^4*x^4*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3*SinhIntegral[ArcSinh[a*x
]] + 81*ArcSinh[a*x]^3*SinhIntegral[3*ArcSinh[a*x]] - 125*ArcSinh[a*x]^3*Si
nhIntegral[5*ArcSinh[a*x]])/(a^5*ArcSinh[a*x]^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^4}{\mathrm{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\mathrm{arsinh}\,(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^4, x)
```

maple [A] time = 0.19, size = 169, normalized size = 1.09

$$\frac{-\frac{\sqrt{a^2x^2+1}}{24\operatorname{arcsinh}(ax)^3}-\frac{ax}{48\operatorname{arcsinh}(ax)^2}-\frac{\sqrt{a^2x^2+1}}{48\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{48}+\frac{\cosh(3\operatorname{arcsinh}(ax))}{16\operatorname{arcsinh}(ax)^3}+\frac{3\sinh(3\operatorname{arcsinh}(ax))}{32\operatorname{arcsinh}(ax)^2}+\frac{9\cosh(3\operatorname{arcsi}}{32\operatorname{arcsinh}}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^4,x)
```

```
[Out] 1/a^5*(-1/24/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-1/48/arcsinh(a*x)^2*a*x-1/48/
arcsinh(a*x)*(a^2*x^2+1)^(1/2)+1/48*Shi(arcsinh(a*x))+1/16/arcsinh(a*x)^3*c
osh(3*arcsinh(a*x))+3/32/arcsinh(a*x)^2*sinh(3*arcsinh(a*x))+9/32/arcsinh(a
*x)*cosh(3*arcsinh(a*x))-27/32*Shi(3*arcsinh(a*x))-1/48/arcsinh(a*x)^3*cosh
(5*arcsinh(a*x))-5/96/arcsinh(a*x)^2*sinh(5*arcsinh(a*x))-25/96/arcsinh(a*x
)*cosh(5*arcsinh(a*x))+125/96*Shi(5*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^15 + 10*a^11*x^13 + 20*a^9*x^11 + 20*a^7*x^9 + 10*a^5*x^7 +
2*a^3*x^5 + 2*(a^8*x^10 + a^6*x^8)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^11 + 9*
a^7*x^9 + 4*a^5*x^7)*(a^2*x^2 + 1)^2 + (25*a^13*x^15 + 125*a^11*x^13 + 250*
a^9*x^11 + 250*a^7*x^9 + 125*a^5*x^7 + 25*a^3*x^5 + (25*a^8*x^10 + 49*a^6*x
^8 + 27*a^4*x^6 + 3*a^2*x^4)*(a^2*x^2 + 1)^(5/2) + (125*a^9*x^11 + 321*a^7*
x^9 + 286*a^5*x^7 + 102*a^3*x^5 + 12*a*x^3)*(a^2*x^2 + 1)^2 + (250*a^10*x^1
2 + 794*a^8*x^10 + 946*a^6*x^8 + 519*a^4*x^6 + 129*a^2*x^4 + 12*x^2)*(a^2*x
^2 + 1)^(3/2) + 2*(125*a^11*x^13 + 473*a^9*x^11 + 696*a^7*x^9 + 497*a^5*x^7
 + 173*a^3*x^5 + 24*a*x^3)*(a^2*x^2 + 1) + (125*a^12*x^14 + 549*a^10*x^12 +
 955*a^8*x^10 + 824*a^6*x^8 + 354*a^4*x^6 + 61*a^2*x^4)*sqrt(a^2*x^2 + 1))*
log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^12 + 13*a^8*x^10 + 11*a^6*x^8
+ 3*a^4*x^6)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^13 + 17*a^9*x^11 + 21*a^7*x^
9 + 11*a^5*x^7 + 2*a^3*x^5)*(a^2*x^2 + 1) + (5*a^13*x^15 + 25*a^11*x^13 + 5
0*a^9*x^11 + 50*a^7*x^9 + 25*a^5*x^7 + 5*a^3*x^5 + (5*a^8*x^10 + 8*a^6*x^8
+ 3*a^4*x^6)*(a^2*x^2 + 1)^(5/2) + (25*a^9*x^11 + 57*a^7*x^9 + 42*a^5*x^7 +
 10*a^3*x^5)*(a^2*x^2 + 1)^2 + (50*a^10*x^12 + 148*a^8*x^10 + 158*a^6*x^8 +
 71*a^4*x^6 + 11*a^2*x^4)*(a^2*x^2 + 1)^(3/2) + 2*(25*a^11*x^13 + 91*a^9*x^
11 + 126*a^7*x^9 + 81*a^5*x^7 + 23*a^3*x^5 + 2*a*x^3)*(a^2*x^2 + 1) + (25*a
^12*x^14 + 108*a^10*x^12 + 183*a^8*x^10 + 151*a^6*x^8 + 60*a^4*x^6 + 9*a^2*
x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^14 + 21*
a^10*x^12 + 34*a^8*x^10 + 26*a^6*x^8 + 9*a^4*x^6 + a^2*x^4)*sqrt(a^2*x^2 +
1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x^2 + 1)^(5/2)*a^8*x^5 + 10*a^9*x^6 + 1
0*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 10*(a
^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^8 + 3*a^9*x
^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 5*(a^12*x^9 + 4*a^10*x^7 + 6*a^8*
x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3)
 + integrate(1/6*(125*a^15*x^16 + 750*a^13*x^14 + 1875*a^11*x^12 + 2500*a^9
*x^10 + 1875*a^7*x^8 + 750*a^5*x^6 + 125*a^3*x^4 + (125*a^9*x^10 + 147*a^7*
x^8 + 27*a^5*x^6 - 3*a^3*x^4)*(a^2*x^2 + 1)^3 + (750*a^10*x^11 + 1485*a^8*x
^9 + 901*a^6*x^7 + 147*a^4*x^5 - 12*a^2*x^3)*(a^2*x^2 + 1)^(5/2) + (1875*a^
11*x^12 + 5220*a^9*x^10 + 5209*a^7*x^8 + 2185*a^5*x^6 + 321*a^3*x^4)*(a^2*x
^2 + 1)^2 + (2500*a^12*x^13 + 8970*a^10*x^11 + 12366*a^8*x^9 + 8143*a^6*x^7
 + 2583*a^4*x^5 + 360*a^2*x^3 + 24*x)*(a^2*x^2 + 1)^(3/2) + (1875*a^13*x^14
 + 8235*a^11*x^12 + 14449*a^9*x^10 + 12834*a^7*x^8 + 6030*a^5*x^6 + 1429*a^
3*x^4 + 144*a*x^2)*(a^2*x^2 + 1) + (750*a^14*x^15 + 3897*a^12*x^13 + 8293*a
^10*x^11 + 9226*a^8*x^9 + 5655*a^6*x^7 + 1819*a^4*x^5 + 244*a^2*x^3)*sqrt(a
```

```
^2*x^2 + 1))/((a^15*x^12 + 6*a^13*x^10 + 15*a^11*x^8 + (a^2*x^2 + 1)^3*a^9*
x^6 + 20*a^9*x^6 + 15*a^7*x^4 + 6*a^5*x^2 + 6*(a^10*x^7 + a^8*x^5)*(a^2*x^2
 + 1)^(5/2) + 15*(a^11*x^8 + 2*a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 2
0*(a^12*x^9 + 3*a^10*x^7 + 3*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a
^13*x^10 + 4*a^11*x^8 + 6*a^9*x^6 + 4*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 6*
(a^14*x^11 + 5*a^12*x^9 + 10*a^10*x^7 + 10*a^8*x^5 + 5*a^6*x^3 + a^4*x)*sqr
t(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{asinh}(a\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asinh(a*x)^4,x)
```

```
[Out] int(x^4/asinh(a*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asinh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**4,x)
```

```
[Out] Integral(x**4/asinh(a*x)**4, x)
```

3.68 $\int \frac{x^3}{\sinh^{-1}(ax)^4} dx$

Optimal. Leaf size=141

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^4}+\frac{4\text{Chi}\left(4\sinh^{-1}(ax)\right)}{3a^4}-\frac{x^2}{2a^2\sinh^{-1}(ax)^2}-\frac{8x^3\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)}-\frac{x^3\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{x\sqrt{a^2x^2+1}}{a^3\sinh^{-1}(ax)}$$

```
[Out] -1/2*x^2/a^2/arcsinh(a*x)^2-2/3*x^4/arcsinh(a*x)^2-1/3*Chi(2*arcsinh(a*x))/
a^4+4/3*Chi(4*arcsinh(a*x))/a^4-1/3*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-
x*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-8/3*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*
x)
```

Rubi [A] time = 0.28, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5667, 5774, 5665, 3301}

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^4}+\frac{4\text{Chi}\left(4\sinh^{-1}(ax)\right)}{3a^4}-\frac{8x^3\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)}-\frac{x^3\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{x^2}{2a^2\sinh^{-1}(ax)^2}-\frac{x\sqrt{a^2x^2+1}}{a^3\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^4,x]
```

```
[Out] -(x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - x^2/(2*a^2*ArcSinh[a*x]^2)
- (2*x^4)/(3*ArcSinh[a*x]^2) - (x*Sqrt[1 + a^2*x^2])/(a^3*ArcSinh[a*x]) - (
8*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]) - CoshIntegral[2*ArcSinh[a*x]]/
(3*a^4) + (4*CoshIntegral[4*ArcSinh[a*x]])/(3*a^4)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^4}\,dx = -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} + \frac{\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx}{a} + \frac{1}{3}(4a)\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx$$
$$= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x^2}{2a^2\sinh^{-1}(ax)^2} - \frac{2x^4}{3\sinh^{-1}(ax)^2} + \frac{8}{3}\int \frac{x^3}{\sinh^{-1}(ax)^2}\,dx + \frac{\int \frac{x}{\sinh^{-1}(ax)^2}\,d}{a^2}$$
$$= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x^2}{2a^2\sinh^{-1}(ax)^2} - \frac{2x^4}{3\sinh^{-1}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{8x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)} + \text{Su}$$
$$= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x^2}{2a^2\sinh^{-1}(ax)^2} - \frac{2x^4}{3\sinh^{-1}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{8x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)} + \text{Cl}$$
$$= -\frac{x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x^2}{2a^2\sinh^{-1}(ax)^2} - \frac{2x^4}{3\sinh^{-1}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\sinh^{-1}(ax)} - \frac{8x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)} - \text{Cl}$$

Mathematica [A] time = 0.42, size = 105, normalized size = 0.74

$$-\frac{\frac{ax\left(2a^2x^2\sqrt{a^2x^2+1}+ax\left(4a^2x^2+3\right)\sinh^{-1}(ax)+2\sqrt{a^2x^2+1}\left(8a^2x^2+3\right)\sinh^{-1}(ax)^2\right)}{\sinh^{-1}(ax)^3} + 2\text{Chi}\left(2\sinh^{-1}(ax)\right) - 8\text{Chi}\left(4\sinh^{-1}(ax)\right)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcSinh[a*x]^4,x]
```

```
[Out] -1/6*((a*x*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + a*x*(3 + 4*a^2*x^2)*ArcSinh[a*x]
+ 2*Sqrt[1 + a^2*x^2]*(3 + 8*a^2*x^2)*ArcSinh[a*x]^2))/ArcSinh[a*x]^3 + 2*C
oshIntegral[2*ArcSinh[a*x]] - 8*CoshIntegral[4*ArcSinh[a*x]])/a^4
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\text{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.18, size = 114, normalized size = 0.81

$$\frac{\frac{\sinh(2\,\text{arcsinh}(ax))}{12\,\text{arcsinh}(ax)^3} + \frac{\cosh(2\,\text{arcsinh}(ax))}{12\,\text{arcsinh}(ax)^2} + \frac{\sinh(2\,\text{arcsinh}(ax))}{6\,\text{arcsinh}(ax)} - \frac{X(2\,\text{arcsinh}(ax))}{3} - \frac{\sinh(4\,\text{arcsinh}(ax))}{24\,\text{arcsinh}(ax)^3} - \frac{\cosh(4\,\text{arcsinh}(ax))}{12\,\text{arcsinh}(ax)^2} - \frac{\sinh(4\,\text{ar}}{3\,\text{arcs}}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^4,x)
```

```
[Out] 1/a^4*(1/12/arcsinh(a*x)^3*sinh(2*arcsinh(a*x))+1/12/arcsinh(a*x)^2*cosh(2*
arcsinh(a*x))+1/6/arcsinh(a*x)*sinh(2*arcsinh(a*x))-1/3*Chi(2*arcsinh(a*x))
-1/24/arcsinh(a*x)^3*sinh(4*arcsinh(a*x))-1/12/arcsinh(a*x)^2*cosh(4*arcsin
h(a*x))-1/3/arcsinh(a*x)*sinh(4*arcsinh(a*x))+4/3*Chi(4*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^14 + 10*a^11*x^12 + 20*a^9*x^10 + 20*a^7*x^8 + 10*a^5*x^6 +
2*a^3*x^4 + 2*(a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^10 + 9*a
^7*x^8 + 4*a^5*x^6)*(a^2*x^2 + 1)^2 + (16*a^13*x^14 + 80*a^11*x^12 + 160*a^
9*x^10 + 160*a^7*x^8 + 80*a^5*x^6 + 16*a^3*x^4 + 4*(4*a^8*x^9 + 7*a^6*x^7 +
 3*a^4*x^5)*(a^2*x^2 + 1)^(5/2) + (80*a^9*x^10 + 192*a^7*x^8 + 154*a^5*x^6
+ 45*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1)^2 + (160*a^10*x^11 + 488*a^8*x^9 + 55
0*a^6*x^7 + 279*a^4*x^5 + 63*a^2*x^3 + 6*x)*(a^2*x^2 + 1)^(3/2) + (160*a^11
*x^12 + 592*a^9*x^10 + 846*a^7*x^8 + 583*a^5*x^6 + 196*a^3*x^4 + 27*a*x^2)*
(a^2*x^2 + 1) + (80*a^12*x^13 + 348*a^10*x^11 + 598*a^8*x^9 + 509*a^6*x^7 +
 216*a^4*x^5 + 37*a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^
2 + 4*(5*a^10*x^11 + 13*a^8*x^9 + 11*a^6*x^7 + 3*a^4*x^5)*(a^2*x^2 + 1)^(3/
2) + 4*(5*a^11*x^12 + 17*a^9*x^10 + 21*a^7*x^8 + 11*a^5*x^6 + 2*a^3*x^4)*(a
^2*x^2 + 1) + (4*a^13*x^14 + 20*a^11*x^12 + 40*a^9*x^10 + 40*a^7*x^8 + 20*a
^5*x^6 + 4*a^3*x^4 + 2*(2*a^8*x^9 + 3*a^6*x^7 + a^4*x^5)*(a^2*x^2 + 1)^(5/2
) + (20*a^9*x^10 + 44*a^7*x^8 + 31*a^5*x^6 + 7*a^3*x^4)*(a^2*x^2 + 1)^2 + (
40*a^10*x^11 + 116*a^8*x^9 + 121*a^6*x^7 + 53*a^4*x^5 + 8*a^2*x^3)*(a^2*x^2
 + 1)^(3/2) + (40*a^11*x^12 + 144*a^9*x^10 + 197*a^7*x^8 + 125*a^5*x^6 + 35
*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1) + (20*a^12*x^13 + 86*a^10*x^11 + 145*a^8*
x^9 + 119*a^6*x^7 + 47*a^4*x^5 + 7*a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sq
rt(a^2*x^2 + 1)) + 2*(5*a^12*x^13 + 21*a^10*x^11 + 34*a^8*x^9 + 26*a^6*x^7
+ 9*a^4*x^5 + a^2*x^3)*sqrt(a^2*x^2 + 1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x
^2 + 1)^(5/2)*a^8*x^5 + 10*a^9*x^6 + 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 +
a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 10*(a^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x
^2 + 1)^(3/2) + 10*(a^11*x^8 + 3*a^9*x^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 +
1) + 5*(a^12*x^9 + 4*a^10*x^7 + 6*a^8*x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2
 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate(1/6*(64*a^15*x^15 + 384*
a^13*x^13 + 960*a^11*x^11 + 1280*a^9*x^9 + 960*a^7*x^7 + 384*a^5*x^5 + 64*a
^3*x^3 + 8*(8*a^9*x^9 + 7*a^7*x^7)*(a^2*x^2 + 1)^3 + (384*a^10*x^10 + 664*a
^8*x^8 + 308*a^6*x^6 + 12*a^4*x^4 - 9*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + 2*(480
*a^11*x^11 + 1240*a^9*x^9 + 1096*a^7*x^7 + 360*a^5*x^5 + 15*a^3*x^3 - 9*a*x
)*(a^2*x^2 + 1)^2 + 2*(640*a^12*x^12 + 2200*a^10*x^10 + 2844*a^8*x^8 + 1684
*a^6*x^6 + 433*a^4*x^4 + 36*a^2*x^2 + 3)*(a^2*x^2 + 1)^(3/2) + 2*(480*a^13*
x^13 + 2060*a^11*x^11 + 3496*a^9*x^9 + 2952*a^7*x^7 + 1283*a^5*x^5 + 274*a^
3*x^3 + 27*a*x)*(a^2*x^2 + 1) + (384*a^14*x^14 + 1976*a^12*x^12 + 4148*a^10
*x^10 + 4524*a^8*x^8 + 2699*a^6*x^6 + 842*a^4*x^4 + 111*a^2*x^2)*sqrt(a^2*x
^2 + 1))/((a^15*x^12 + 6*a^13*x^10 + 15*a^11*x^8 + (a^2*x^2 + 1)^3*a^9*x^6
+ 20*a^9*x^6 + 15*a^7*x^4 + 6*a^5*x^2 + 6*(a^10*x^7 + a^8*x^5)*(a^2*x^2 + 1
)^(5/2) + 15*(a^11*x^8 + 2*a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 20*(a
^12*x^9 + 3*a^10*x^7 + 3*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*
x^10 + 4*a^11*x^8 + 6*a^9*x^6 + 4*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 6*(a^1
4*x^11 + 5*a^12*x^9 + 10*a^10*x^7 + 10*a^8*x^5 + 5*a^6*x^3 + a^4*x)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{asinh}(a\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asinh(a*x)^4,x)
```

```
[Out] int(x^3/asinh(a*x)^4, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asinh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**4,x)
```

```
[Out] Integral(x**3/asinh(a*x)**4, x)
```

3.69 $\int \frac{x^2}{\sinh^{-1}(ax)^4} \, dx$

Optimal. Leaf size=138

$$-\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{24a^3}+\frac{9\text{Shi}\left(3\sinh^{-1}(ax)\right)}{8a^3}-\frac{3x^2\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)}-\frac{x^2\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{x}{3a^2\sinh^{-1}(ax)^2}-\frac{\sqrt{a^2x^2+1}}{3a^3\sinh^{-1}(ax)}-$$

```
[Out] -1/3*x/a^2/arcsinh(a*x)^2-1/2*x^3/arcsinh(a*x)^2-1/24*Shi(arcsinh(a*x))/a^3
+9/8*Shi(3*arcsinh(a*x))/a^3-1/3*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-1/3
*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-3/2*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x
)
```

Rubi [A] time = 0.31, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5667, 5774, 5665, 3298, 5655, 5779}

$$-\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{24a^3}+\frac{9\text{Shi}\left(3\sinh^{-1}(ax)\right)}{8a^3}-\frac{3x^2\sqrt{a^2x^2+1}}{2a\sinh^{-1}(ax)}-\frac{x^2\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{\sqrt{a^2x^2+1}}{3a^3\sinh^{-1}(ax)}-\frac{x}{3a^2\sinh^{-1}(ax)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^4,x]
```

```
[Out] -(x^2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - x/(3*a^2*ArcSinh[a*x]^2) -
x^3/(2*ArcSinh[a*x]^2) - Sqrt[1 + a^2*x^2]/(3*a^3*ArcSinh[a*x]) - (3*x^2*Sq
rt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]) - SinhIntegral[ArcSinh[a*x]]/(24*a^3) +
 (9*SinhIntegral[3*ArcSinh[a*x]])/(8*a^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^4}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} + \frac{2\int \frac{x}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}\,dx}{3a} + a\int \frac{x^3}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}\,dx$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{3a^2\sinh^{-1}(ax)^2} - \frac{x^3}{2\sinh^{-1}(ax)^2} + \frac{3}{2}\int \frac{x^2}{\sinh^{-1}(ax)^2}\,dx + \frac{\int \frac{1}{\sinh^{-1}(ax)^2}\,d}{3a^2}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{3a^2\sinh^{-1}(ax)^2} - \frac{x^3}{2\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\sinh^{-1}(ax)} - \frac{3x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)} + \frac{3}{}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{3a^2\sinh^{-1}(ax)^2} - \frac{x^3}{2\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\sinh^{-1}(ax)} - \frac{3x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)} + \frac{S}{}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{3a^2\sinh^{-1}(ax)^2} - \frac{x^3}{2\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\sinh^{-1}(ax)} - \frac{3x^2\sqrt{1+a^2x^2}}{2a\sinh^{-1}(ax)} - \frac{S}{}$$

Mathematica [A] time = 0.33, size = 99, normalized size = 0.72

$$-\frac{\frac{4\left(2a^2x^2\sqrt{a^2x^2+1}+ax\left(3a^2x^2+2\right)\sinh^{-1}(ax)+\sqrt{a^2x^2+1}\left(9a^2x^2+2\right)\sinh^{-1}(ax)^2\right)}{\sinh^{-1}(ax)^3}+\text{Shi}\left(\sinh^{-1}(ax)\right)-27\text{Shi}\left(3\sinh^{-1}(ax)\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a*x]^4,x]
```

```
[Out] -1/24*((4*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + a*x*(2 + 3*a^2*x^2)*ArcSinh[a*x] +
 Sqrt[1 + a^2*x^2]*(2 + 9*a^2*x^2)*ArcSinh[a*x]^2))/ArcSinh[a*x]^3 + SinhIn
tegral[ArcSinh[a*x]] - 27*SinhIntegral[3*ArcSinh[a*x]])/a^3
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\text{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^4, x)
```

maple [A] time = 0.14, size = 115, normalized size = 0.83

$$\frac{\frac{\sqrt{a^2x^2+1}}{12\operatorname{arcsinh}(ax)^3}+\frac{ax}{24\operatorname{arcsinh}(ax)^2}+\frac{\sqrt{a^2x^2+1}}{24\operatorname{arcsinh}(ax)}-\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{24}-\frac{\cosh(3\operatorname{arcsinh}(ax))}{12\operatorname{arcsinh}(ax)^3}-\frac{\sinh(3\operatorname{arcsinh}(ax))}{8\operatorname{arcsinh}(ax)^2}-\frac{3\cosh\left(3\operatorname{arcsinh}(a\right.}{8\operatorname{arcsinh}(ax)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^4,x)
```

```
[Out] 1/a^3*(1/12/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+1/24/arcsinh(a*x)^2*a*x+1/24/a
rcsinh(a*x)*(a^2*x^2+1)^(1/2)-1/24*Shi(arcsinh(a*x))-1/12/arcsinh(a*x)^3*co
sh(3*arcsinh(a*x))-1/8/arcsinh(a*x)^2*sinh(3*arcsinh(a*x))-3/8/arcsinh(a*x)
*cosh(3*arcsinh(a*x))+9/8*Shi(3*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (9*a^13*x^13 + 45*a^11*x^11 + 90*a^9*x^
9 + 90*a^7*x^7 + 45*a^5*x^5 + 9*a^3*x^3 + (9*a^8*x^8 + 13*a^6*x^6 + 3*a^4*x
^4 - a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (45*a^9*x^9 + 97*a^7*x^7 + 64*a^5*x^5 +
 10*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^2 + (90*a^10*x^10 + 258*a^8*x^8 + 264*a^
6*x^6 + 113*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2*x^2 + 1)^(3/2) + 2*(45*a^11*x^11
 + 161*a^9*x^9 + 219*a^7*x^7 + 141*a^5*x^5 + 44*a^3*x^3 + 6*a*x)*(a^2*x^2 +
 1) + (45*a^12*x^12 + 193*a^10*x^10 + 325*a^8*x^8 + 270*a^6*x^6 + 112*a^4*x
^4 + 19*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a
^10*x^10 + 13*a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*
a^11*x^11 + 17*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1)
 + (3*a^13*x^13 + 15*a^11*x^11 + 30*a^9*x^9 + 30*a^7*x^7 + 15*a^5*x^5 + 3*a
^3*x^3 + (3*a^8*x^8 + 4*a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (15*a^9*x^
9 + 31*a^7*x^7 + 20*a^5*x^5 + 4*a^3*x^3)*(a^2*x^2 + 1)^2 + (30*a^10*x^10 +
84*a^8*x^8 + 84*a^6*x^6 + 35*a^4*x^4 + 5*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 2*(
15*a^11*x^11 + 53*a^9*x^9 + 71*a^7*x^7 + 44*a^5*x^5 + 12*a^3*x^3 + a*x)*(a^
2*x^2 + 1) + (15*a^12*x^12 + 64*a^10*x^10 + 107*a^8*x^8 + 87*a^6*x^6 + 34*a
^4*x^4 + 5*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*
a^12*x^12 + 21*a^10*x^10 + 34*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*s
qrt(a^2*x^2 + 1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x^2 + 1)^(5/2)*a^8*x^5 +
10*a^9*x^6 + 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2
 + a^3 + 10*(a^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^11
*x^8 + 3*a^9*x^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 5*(a^12*x^9 + 4*a^1
0*x^7 + 6*a^8*x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^
2*x^2 + 1))^3) + integrate(1/6*(27*a^14*x^14 + 162*a^12*x^12 + 405*a^10*x^1
0 + 540*a^8*x^8 + 405*a^6*x^6 + 162*a^4*x^4 + (27*a^8*x^8 + 13*a^6*x^6 - 3*
a^4*x^4 + 3*a^2*x^2)*(a^2*x^2 + 1)^3 + 27*a^2*x^2 + (162*a^9*x^9 + 227*a^7*
```

```
x^7 + 63*a^5*x^5 - 3*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1)^(5/2) + (405*a^10*x^10
+ 940*a^8*x^8 + 687*a^6*x^6 + 143*a^4*x^4 - 21*a^2*x^2 - 12)*(a^2*x^2 + 1)^
2 + (540*a^11*x^11 + 1750*a^9*x^9 + 2058*a^7*x^7 + 1017*a^5*x^5 + 145*a^3*x
^3 - 24*a*x)*(a^2*x^2 + 1)^(3/2) + (405*a^12*x^12 + 1685*a^10*x^10 + 2727*a
^8*x^8 + 2118*a^6*x^6 + 782*a^4*x^4 + 123*a^2*x^2 + 12)*(a^2*x^2 + 1) + (16
2*a^13*x^13 + 823*a^11*x^11 + 1695*a^9*x^9 + 1790*a^7*x^7 + 1015*a^5*x^5 +
297*a^3*x^3 + 38*a*x)*sqrt(a^2*x^2 + 1))/((a^14*x^12 + 6*a^12*x^10 + 15*a^1
0*x^8 + (a^2*x^2 + 1)^3*a^8*x^6 + 20*a^8*x^6 + 15*a^6*x^4 + 6*a^4*x^2 + 6*(
a^9*x^7 + a^7*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^10*x^8 + 2*a^8*x^6 + a^6*x^4
)*(a^2*x^2 + 1)^2 + 20*(a^11*x^9 + 3*a^9*x^7 + 3*a^7*x^5 + a^5*x^3)*(a^2*x^
2 + 1)^(3/2) + 15*(a^12*x^10 + 4*a^10*x^8 + 6*a^8*x^6 + 4*a^6*x^4 + a^4*x^2
)*(a^2*x^2 + 1) + a^2 + 6*(a^13*x^11 + 5*a^11*x^9 + 10*a^9*x^7 + 10*a^7*x^5
 + 5*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\mathrm{asinh}\left(a\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a*x)^4,x)
```

```
[Out] int(x^2/asinh(a*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**4,x)
```

```
[Out] Integral(x**2/asinh(a*x)**4, x)
```

3.70 $\int \frac{x}{\sinh^{-1}(ax)^4} \, dx$

Optimal. Leaf size=95

$$\frac{2\text{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^2}-\frac{2x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)}-\frac{x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{1}{6a^2\sinh^{-1}(ax)^2}-\frac{x^2}{3\sinh^{-1}(ax)^2}$$

```
[Out] -1/6/a^2/arcsinh(a*x)^2-1/3*x^2/arcsinh(a*x)^2+2/3*Chi(2*arcsinh(a*x))/a^2-
1/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-2/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(
a*x)
```

Rubi [A] time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5667, 5774, 5665, 3301, 5675}

$$\frac{2\text{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^2}-\frac{2x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)}-\frac{x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}-\frac{1}{6a^2\sinh^{-1}(ax)^2}-\frac{x^2}{3\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^4,x]
```

```
[Out] -(x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^3) - 1/(6*a^2*ArcSinh[a*x]^2) - x^
2/(3*ArcSinh[a*x]^2) - (2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]) + (2*Cosh
Integral[2*ArcSinh[a*x]])/(3*a^2)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
```

```
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(ax)^4}\,dx = -\frac{x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx}{3a} + \frac{1}{3}(2a)\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx$$
$$= -\frac{x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{1}{6a^2\sinh^{-1}(ax)^2} - \frac{x^2}{3\sinh^{-1}(ax)^2} + \frac{2}{3}\int \frac{x}{\sinh^{-1}(ax)^2}\,dx$$
$$= -\frac{x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{1}{6a^2\sinh^{-1}(ax)^2} - \frac{x^2}{3\sinh^{-1}(ax)^2} - \frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{x}\right.}{3a}$$
$$= -\frac{x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{1}{6a^2\sinh^{-1}(ax)^2} - \frac{x^2}{3\sinh^{-1}(ax)^2} - \frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)} + \frac{2\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{3a^2}$$

Mathematica [A] time = 0.13, size = 84, normalized size = 0.88

$$-\frac{2ax\sqrt{a^2x^2+1}+4ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2+\left(2a^2x^2+1\right)\sinh^{-1}(ax)-4\sinh^{-1}(ax)^3\mathrm{Chi}\left(2\sinh^{-1}(ax)\right)}{6a^2\sinh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a*x]^4,x]
[Out] -1/6*(2*a*x*Sqrt[1 + a^2*x^2] + (1 + 2*a^2*x^2)*ArcSinh[a*x] + 4*a*x*Sqrt[1
 + a^2*x^2]*ArcSinh[a*x]^2 - 4*ArcSinh[a*x]^3*CoshIntegral[2*ArcSinh[a*x]])
/(a^2*ArcSinh[a*x]^3)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{\mathrm{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="fricas")
[Out] integral(x/arcsinh(a*x)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{arsinh}\,(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="giac")
[Out] integrate(x/arcsinh(a*x)^4, x)
```

maple [A] time = 0.17, size = 60, normalized size = 0.63

$$\frac{-\frac{\sinh(2\,\mathrm{arcsinh}(ax))}{6\,\mathrm{arcsinh}(ax)^3} - \frac{\cosh(2\,\mathrm{arcsinh}(ax))}{6\,\mathrm{arcsinh}(ax)^2} - \frac{\sinh(2\,\mathrm{arcsinh}(ax))}{3\,\mathrm{arcsinh}(ax)} + \frac{2\mathrm{X}(2\,\mathrm{arcsinh}(ax))}{3}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^4,x)
```

```
[Out] 1/a^2*(-1/6/arcsinh(a*x)^3*sinh(2*arcsinh(a*x))-1/6/arcsinh(a*x)^2*cosh(2*a
rcsinh(a*x))-1/3/arcsinh(a*x)*sinh(2*arcsinh(a*x))+2/3*Chi(2*arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^12*x^12 + 10*a^10*x^10 + 20*a^8*x^8 + 20*a^6*x^6 + 10*a^4*x^4 + 2
*a^2*x^2 + 2*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^8*x^8 + 9*a^6
*x^6 + 4*a^4*x^4)*(a^2*x^2 + 1)^2 + (4*a^12*x^12 + 20*a^10*x^10 + 40*a^8*x^
8 + 40*a^6*x^6 + 20*a^4*x^4 + 4*a^2*x^2 + 4*(a^7*x^7 + a^5*x^5)*(a^2*x^2 +
1)^(5/2) + (20*a^8*x^8 + 36*a^6*x^6 + 16*a^4*x^4 - 3*a^2*x^2 - 3)*(a^2*x^2
+ 1)^2 + (40*a^9*x^9 + 104*a^7*x^7 + 88*a^5*x^5 + 21*a^3*x^3 - 3*a*x)*(a^2*
x^2 + 1)^(3/2) + (40*a^10*x^10 + 136*a^8*x^8 + 168*a^6*x^6 + 91*a^4*x^4 + 2
2*a^2*x^2 + 3)*(a^2*x^2 + 1) + (20*a^11*x^11 + 84*a^9*x^9 + 136*a^7*x^7 + 1
07*a^5*x^5 + 42*a^3*x^3 + 7*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
+ 1))^2 + 4*(5*a^9*x^9 + 13*a^7*x^7 + 11*a^5*x^5 + 3*a^3*x^3)*(a^2*x^2 + 1)
^(3/2) + 4*(5*a^10*x^10 + 17*a^8*x^8 + 21*a^6*x^6 + 11*a^4*x^4 + 2*a^2*x^2)
*(a^2*x^2 + 1) + (2*a^12*x^12 + 10*a^10*x^10 + 20*a^8*x^8 + 20*a^6*x^6 + 10
*a^4*x^4 + 2*a^2*x^2 + 2*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + (10*a^8*
x^8 + 18*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1)^2 + (20*a^9*x^9 + 52*
a^7*x^7 + 47*a^5*x^5 + 17*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1)^(3/2) + (20*a^10*x
^10 + 68*a^8*x^8 + 87*a^6*x^6 + 51*a^4*x^4 + 13*a^2*x^2 + 1)*(a^2*x^2 + 1)
+ (10*a^11*x^11 + 42*a^9*x^9 + 69*a^7*x^7 + 55*a^5*x^5 + 21*a^3*x^3 + 3*a*x
)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^11*x^11 + 21*a^9
*x^9 + 34*a^7*x^7 + 26*a^5*x^5 + 9*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^12
*x^10 + 5*a^10*x^8 + (a^2*x^2 + 1)^(5/2)*a^7*x^5 + 10*a^8*x^6 + 10*a^6*x^4
+ 5*a^4*x^2 + 5*(a^8*x^6 + a^6*x^4)*(a^2*x^2 + 1)^2 + 10*(a^9*x^7 + 2*a^7*x
^5 + a^5*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^10*x^8 + 3*a^8*x^6 + 3*a^6*x^4 +
a^4*x^2)*(a^2*x^2 + 1) + a^2 + 5*(a^11*x^9 + 4*a^9*x^7 + 6*a^7*x^5 + 4*a^5*
x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate
(1/6*(8*a^13*x^13 + 48*a^11*x^11 + 120*a^9*x^9 + 8*(a^2*x^2 + 1)^3*a^7*x^7
+ 160*a^7*x^7 + 120*a^5*x^5 + 48*a^3*x^3 + (48*a^8*x^8 + 48*a^6*x^6 + 4*a^4
*x^4 + 12*a^2*x^2 + 15)*(a^2*x^2 + 1)^(5/2) + 8*(15*a^9*x^9 + 30*a^7*x^7 +
17*a^5*x^5 + 5*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1)^2 + 2*(80*a^10*x^10 + 240*a^8
*x^8 + 252*a^6*x^6 + 104*a^4*x^4 + 3*a^2*x^2 - 9)*(a^2*x^2 + 1)^(3/2) + 8*(
15*a^11*x^11 + 60*a^9*x^9 + 92*a^7*x^7 + 63*a^5*x^5 + 15*a^3*x^3 - a*x)*(a^
2*x^2 + 1) + 8*a*x + (48*a^12*x^12 + 240*a^10*x^10 + 484*a^8*x^8 + 484*a^6*
x^6 + 243*a^4*x^4 + 58*a^2*x^2 + 7)*sqrt(a^2*x^2 + 1))/((a^13*x^12 + 6*a^11
*x^10 + 15*a^9*x^8 + (a^2*x^2 + 1)^3*a^7*x^6 + 20*a^7*x^6 + 15*a^5*x^4 + 6*
a^3*x^2 + 6*(a^8*x^7 + a^6*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^9*x^8 + 2*a^7*x
^6 + a^5*x^4)*(a^2*x^2 + 1)^2 + 20*(a^10*x^9 + 3*a^8*x^7 + 3*a^6*x^5 + a^4*
x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^11*x^10 + 4*a^9*x^8 + 6*a^7*x^6 + 4*a^5*x^
4 + a^3*x^2)*(a^2*x^2 + 1) + 6*(a^12*x^11 + 5*a^10*x^9 + 10*a^8*x^7 + 10*a^
6*x^5 + 5*a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 +
1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\mathrm{asinh}\left(a\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a*x)^4,x)
```

```
[Out] int(x/asinh(a*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**4,x)
```

```
[Out] Integral(x/asinh(a*x)**4, x)
```

3.71 $\int \frac{1}{\sinh^{-1}(ax)^4} dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{a^2x^2+1}}{6a\sinh^{-1}(ax)}-\frac{\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}+\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{6a}-\frac{x}{6\sinh^{-1}(ax)^2}$$

```
[Out] -1/6*x/arcsinh(a*x)^2+1/6*Shi(arcsinh(a*x))/a-1/3*(a^2*x^2+1)^(1/2)/a/arcsi
nh(a*x)^3-1/6*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] time = 0.15, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5655, 5774, 5779, 3298}

$$-\frac{\sqrt{a^2x^2+1}}{6a\sinh^{-1}(ax)}-\frac{\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^3}+\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{6a}-\frac{x}{6\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-4),x]
```

```
[Out] -Sqrt[1 + a^2*x^2]/(3*a*ArcSinh[a*x]^3) - x/(6*ArcSinh[a*x]^2) - Sqrt[1 + a
^2*x^2]/(6*a*ArcSinh[a*x]) + SinhIntegral[ArcSinh[a*x]]/(6*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{-1}(ax)^4}\,dx &= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} + \frac{1}{3}a\int \frac{x}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^3}\,dx \\ &= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{6\sinh^{-1}(ax)^2} + \frac{1}{6}\int \frac{1}{\sinh^{-1}(ax)^2}\,dx \\ &= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{6\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)} + \frac{1}{6}a\int \frac{x}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)}\,dx \\ &= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{6\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{6a} \\ &= -\frac{\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^3} - \frac{x}{6\sinh^{-1}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{6a\sinh^{-1}(ax)} + \frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{6a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 0.91

$$-\frac{2\sqrt{a^2x^2+1} + \sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^2 + \sinh^{-1}(ax)^3\left(-\text{Shi}\left(\sinh^{-1}(ax)\right)\right) + ax\sinh^{-1}(ax)}{6a\sinh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(-4),x]
[Out] -1/6*(2*Sqrt[1 + a^2*x^2] + a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a*
x]^2 - ArcSinh[a*x]^3*SinhIntegral[ArcSinh[a*x]])/(a*ArcSinh[a*x]^3)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\text{arsinh}\,(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^(-4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\text{arsinh}\,(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(-4), x)
```

maple [A] time = 0.10, size = 61, normalized size = 0.80

$$\frac{-\frac{\sqrt{a^2x^2+1}}{3\,\text{arcsinh}(ax)^3} - \frac{ax}{6\,\text{arcsinh}(ax)^2} - \frac{\sqrt{a^2x^2+1}}{6\,\text{arcsinh}(ax)} + \frac{\text{Shi}(\text{arcsinh}(ax))}{6}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^4,x)
```

```
[Out] 1/a*(-1/3/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-1/6/arcsinh(a*x)^2*a*x-1/6/arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)+1/6*Shi(arcsinh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^11*x^11 + 10*a^9*x^9 + 20*a^7*x^7 + 20*a^5*x^5 + 10*a^3*x^3 + 2*(
a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^7*x^7 + 9*a^5*x^5 + 4*a^3*x
^3)*(a^2*x^2 + 1)^2 + (a^11*x^11 + 5*a^9*x^9 + 10*a^7*x^7 + 10*a^5*x^5 + 5*
a^3*x^3 + (a^6*x^6 + a^4*x^4 + 3*a^2*x^2 + 3)*(a^2*x^2 + 1)^(5/2) + (5*a^7*
x^7 + 9*a^5*x^5 + 10*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1)^2 + (10*a^8*x^8 + 26*a^
6*x^6 + 22*a^4*x^4 + 3*a^2*x^2 - 3)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^9*x^9 + 17
*a^7*x^7 + 18*a^5*x^5 + 5*a^3*x^3 - a*x)*(a^2*x^2 + 1) + a*x + (5*a^10*x^10
 + 21*a^8*x^8 + 31*a^6*x^6 + 20*a^4*x^4 + 6*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))
*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^8*x^8 + 13*a^6*x^6 + 11*a^4*x^4 +
3*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^9*x^9 + 17*a^7*x^7 + 21*a^5*x^5 + 1
1*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1) + 2*a*x + (a^11*x^11 + 5*a^9*x^9 + 10*a^7*
x^7 + 10*a^5*x^5 + 5*a^3*x^3 + (a^6*x^6 - a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (5
*a^7*x^7 + 5*a^5*x^5 - 2*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^2 + (10*a^8*x^8 + 2
0*a^6*x^6 + 10*a^4*x^4 - a^2*x^2 - 1)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^9*x^9 +
15*a^7*x^7 + 16*a^5*x^5 + 7*a^3*x^3 + a*x)*(a^2*x^2 + 1) + a*x + (5*a^10*x^
10 + 20*a^8*x^8 + 31*a^6*x^6 + 23*a^4*x^4 + 8*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1
))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^10*x^10 + 21*a^8*x^8 + 34*a^6*x^6
+ 26*a^4*x^4 + 9*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^11*x^10 + 5*a^9*x^8 +
(a^2*x^2 + 1)^(5/2)*a^6*x^5 + 10*a^7*x^6 + 10*a^5*x^4 + 5*a^3*x^2 + 5*(a^7*
x^6 + a^5*x^4)*(a^2*x^2 + 1)^2 + 10*(a^8*x^7 + 2*a^6*x^5 + a^4*x^3)*(a^2*x^
2 + 1)^(3/2) + 10*(a^9*x^8 + 3*a^7*x^6 + 3*a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1)
 + 5*(a^10*x^9 + 4*a^8*x^7 + 6*a^6*x^5 + 4*a^4*x^3 + a^2*x)*sqrt(a^2*x^2 +
1) + a)*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate(1/6*(a^12*x^12 + 6*a^10
*x^10 + 15*a^8*x^8 + 20*a^6*x^6 + 15*a^4*x^4 + (a^6*x^6 - a^4*x^4 - 9*a^2*x
^2 - 15)*(a^2*x^2 + 1)^3 + 6*a^2*x^2 + (6*a^7*x^7 + a^5*x^5 - 31*a^3*x^3 -
33*a*x)*(a^2*x^2 + 1)^(5/2) + (15*a^8*x^8 + 20*a^6*x^6 - 19*a^4*x^4 - 3*a^2
*x^2 + 21)*(a^2*x^2 + 1)^2 + (20*a^9*x^9 + 50*a^7*x^7 + 54*a^5*x^5 + 59*a^3
*x^3 + 35*a*x)*(a^2*x^2 + 1)^(3/2) + (15*a^10*x^10 + 55*a^8*x^8 + 101*a^6*x
^6 + 90*a^4*x^4 + 22*a^2*x^2 - 7)*(a^2*x^2 + 1) + (6*a^11*x^11 + 29*a^9*x^9
 + 65*a^7*x^7 + 66*a^5*x^5 + 23*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^1
2*x^12 + 6*a^10*x^10 + 15*a^8*x^8 + (a^2*x^2 + 1)^3*a^6*x^6 + 20*a^6*x^6 +
15*a^4*x^4 + 6*a^2*x^2 + 6*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^
8*x^8 + 2*a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^2 + 20*(a^9*x^9 + 3*a^7*x^7 + 3*
a^5*x^5 + a^3*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^10*x^10 + 4*a^8*x^8 + 6*a^6*
x^6 + 4*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1) + 6*(a^11*x^11 + 5*a^9*x^9 + 10*a^
7*x^7 + 10*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt
(a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}(a\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a*x)^4,x)
```

```
[Out] int(1/asinh(a*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^{4}{\left(ax \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**4,x)
```

```
[Out] Integral(asinh(a*x)**(-4), x)
```

3.72 $\int \frac{1}{x\sinh^{-1}(ax)^4}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x\sinh^{-1}(ax)^4},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^4,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sinh^{-1}(ax)^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^4),x]
[Out] Defer[Int][1/(x*ArcSinh[a*x]^4), x]
```

Rubi steps

$$\int \frac{1}{x\sinh^{-1}(ax)^4}\,dx = \int \frac{1}{x\sinh^{-1}(ax)^4}\,dx$$

Mathematica [A] time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sinh^{-1}(ax)^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^4),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^4), x]
```

fricas [A] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\,\text{arsinh}\,(ax)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(a*x)^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\text{arsinh}\,(ax)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^4), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^4,x)
```

```
[Out] int(1/x/arcsinh(a*x)^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (4*(a^6*x^6 + 3*a^4*x^4 + 2*a^2*x^2)*(a
^2*x^2 + 1)^(5/2) + (16*a^7*x^7 + 46*a^5*x^5 + 37*a^3*x^3 + 7*a*x)*(a^2*x^2
 + 1)^2 + (24*a^8*x^8 + 66*a^6*x^6 + 59*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2*x^2
+ 1)^(3/2) + (16*a^9*x^9 + 42*a^7*x^7 + 39*a^5*x^5 + 16*a^3*x^3 + 3*a*x)*(a
^2*x^2 + 1) + (4*a^10*x^10 + 10*a^8*x^8 + 9*a^6*x^6 + 4*a^4*x^4 + a^2*x^2)*
sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^10 + 13*a^8
*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^11 + 17*a^
9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1) - (2*(a^6*x^6 +
a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (8*a^7*x^7 + 13*a^5*x^5 + 5*a^3*x^3)*(a^2*x^
2 + 1)^2 + (12*a^8*x^8 + 27*a^6*x^6 + 19*a^4*x^4 + 4*a^2*x^2)*(a^2*x^2 + 1)
^(3/2) + (8*a^9*x^9 + 23*a^7*x^7 + 23*a^5*x^5 + 9*a^3*x^3 + a*x)*(a^2*x^2 +
 1) + (2*a^10*x^10 + 7*a^8*x^8 + 9*a^6*x^6 + 5*a^4*x^4 + a^2*x^2)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^12 + 21*a^10*x^10 + 34
*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^13*x^13
 + 5*a^11*x^11 + (a^2*x^2 + 1)^(5/2)*a^8*x^8 + 10*a^9*x^9 + 10*a^7*x^7 + 5*
a^5*x^5 + a^3*x^3 + 5*(a^9*x^9 + a^7*x^7)*(a^2*x^2 + 1)^2 + 10*(a^10*x^10 +
 2*a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^11 + 3*a^9*x^9 + 3*a
^7*x^7 + a^5*x^5)*(a^2*x^2 + 1) + 5*(a^12*x^12 + 4*a^10*x^10 + 6*a^8*x^8 +
4*a^6*x^6 + a^4*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) - i
ntegrate(1/6*(8*(a^7*x^7 + 6*a^5*x^5 + 6*a^3*x^3)*(a^2*x^2 + 1)^3 + (40*a^8
*x^8 + 204*a^6*x^6 + 228*a^4*x^4 + 57*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + 2*(40*
a^9*x^9 + 168*a^7*x^7 + 200*a^5*x^5 + 87*a^3*x^3 + 15*a*x)*(a^2*x^2 + 1)^2
+ 2*(40*a^10*x^10 + 132*a^8*x^8 + 156*a^6*x^6 + 91*a^4*x^4 + 30*a^2*x^2 + 3
)*(a^2*x^2 + 1)^(3/2) + 2*(20*a^11*x^11 + 48*a^9*x^9 + 48*a^7*x^7 + 35*a^5*
x^5 + 18*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1) + (8*a^12*x^12 + 12*a^10*x^10 + 4*a
^8*x^8 + 5*a^6*x^6 + 6*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^15*x^16 +
6*a^13*x^14 + 15*a^11*x^12 + (a^2*x^2 + 1)^3*a^9*x^10 + 20*a^9*x^10 + 15*a^
7*x^8 + 6*a^5*x^6 + a^3*x^4 + 6*(a^10*x^11 + a^8*x^9)*(a^2*x^2 + 1)^(5/2) +
 15*(a^11*x^12 + 2*a^9*x^10 + a^7*x^8)*(a^2*x^2 + 1)^2 + 20*(a^12*x^13 + 3*
a^10*x^11 + 3*a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*x^14 + 4*a^
11*x^12 + 6*a^9*x^10 + 4*a^7*x^8 + a^5*x^6)*(a^2*x^2 + 1) + 6*(a^14*x^15 +
5*a^12*x^13 + 10*a^10*x^11 + 10*a^8*x^9 + 5*a^6*x^7 + a^4*x^5)*sqrt(a^2*x^2
 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{asinh}(a\,x)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)^4),x)
```

```
[Out] int(1/(x*asinh(a*x)^4), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^{4}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**4,x)
```

```
[Out] Integral(1/(x*asinh(a*x)**4), x)
```

3.73 $\int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2 \sinh^{-1}(ax)^4}, x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)^4,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*ArcSinh[a*x]^4),x]
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^4), x]
```

Rubi steps

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx = \int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx$$

Mathematica [A] time = 9.34, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sinh^{-1}(ax)^4} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*ArcSinh[a*x]^4),x]
[Out] Integrate[1/(x^2*ArcSinh[a*x]^4), x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x^2 \text{arsinh}(ax)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="fricas")
[Out] integral(1/(x^2*arcsinh(a*x)^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \text{arsinh}(ax)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="giac")
[Out] integrate(1/(x^2*arcsinh(a*x)^4), x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)^4,x)
```

```
[Out] int(1/x^2/arcsinh(a*x)^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (a^13*x^13 + 5*a^11*x^11 + 10*a^9*x^9 +
 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3 + (a^8*x^8 + 13*a^6*x^6 + 27*a^4*x^4 + 15
*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (5*a^9*x^9 + 57*a^7*x^7 + 124*a^5*x^5 + 90*
a^3*x^3 + 18*a*x)*(a^2*x^2 + 1)^2 + (10*a^10*x^10 + 98*a^8*x^8 + 220*a^6*x^
6 + 189*a^4*x^4 + 63*a^2*x^2 + 6)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^11*x^11 + 41
*a^9*x^9 + 93*a^7*x^7 + 89*a^5*x^5 + 38*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1) + (5
*a^12*x^12 + 33*a^10*x^10 + 73*a^8*x^8 + 74*a^6*x^6 + 36*a^4*x^4 + 7*a^2*x^
2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^10 + 13*
a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^11 + 17
*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1) - (a^13*x^13
+ 5*a^11*x^11 + 10*a^9*x^9 + 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3 + (a^8*x^8 +
4*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (5*a^9*x^9 + 21*a^7*x^7 + 24*a
^5*x^5 + 8*a^3*x^3)*(a^2*x^2 + 1)^2 + (10*a^10*x^10 + 44*a^8*x^8 + 64*a^6*x
^6 + 37*a^4*x^4 + 7*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^11*x^11 + 23*a^9*
x^9 + 39*a^7*x^7 + 30*a^5*x^5 + 10*a^3*x^3 + a*x)*(a^2*x^2 + 1) + (5*a^12*x
^12 + 24*a^10*x^10 + 45*a^8*x^8 + 41*a^6*x^6 + 18*a^4*x^4 + 3*a^2*x^2)*sqrt
(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^12 + 21*a^10*x^10
 + 34*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^13
*x^14 + 5*a^11*x^12 + (a^2*x^2 + 1)^(5/2)*a^8*x^9 + 10*a^9*x^10 + 10*a^7*x^
8 + 5*a^5*x^6 + a^3*x^4 + 5*(a^9*x^10 + a^7*x^8)*(a^2*x^2 + 1)^2 + 10*(a^10
*x^11 + 2*a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^12 + 3*a^9*x^
10 + 3*a^7*x^8 + a^5*x^6)*(a^2*x^2 + 1) + 5*(a^12*x^13 + 4*a^10*x^11 + 6*a^
8*x^9 + 4*a^6*x^7 + a^4*x^5)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)
)^3) - integrate(1/6*(a^15*x^15 + 6*a^13*x^13 + 15*a^11*x^11 + 20*a^9*x^9 +
 15*a^7*x^7 + 6*a^5*x^5 + a^3*x^3 + (a^9*x^9 + 39*a^7*x^7 + 135*a^5*x^5 + 1
05*a^3*x^3)*(a^2*x^2 + 1)^3 + (6*a^10*x^10 + 201*a^8*x^8 + 677*a^6*x^6 + 66
3*a^4*x^4 + 174*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (15*a^11*x^11 + 420*a^9*x^9
+ 1373*a^7*x^7 + 1565*a^5*x^5 + 705*a^3*x^3 + 108*a*x)*(a^2*x^2 + 1)^2 + (2
0*a^12*x^12 + 450*a^10*x^10 + 1422*a^8*x^8 + 1787*a^6*x^6 + 1059*a^4*x^4 +
288*a^2*x^2 + 24)*(a^2*x^2 + 1)^(3/2) + (15*a^13*x^13 + 255*a^11*x^11 + 773
*a^9*x^9 + 1026*a^7*x^7 + 714*a^5*x^5 + 257*a^3*x^3 + 36*a*x)*(a^2*x^2 + 1)
 + (6*a^14*x^14 + 69*a^12*x^12 + 197*a^10*x^10 + 266*a^8*x^8 + 201*a^6*x^6
+ 83*a^4*x^4 + 14*a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^15*x^17 + 6*a^13*x^15 + 1
5*a^11*x^13 + (a^2*x^2 + 1)^3*a^9*x^11 + 20*a^9*x^11 + 15*a^7*x^9 + 6*a^5*x
^7 + a^3*x^5 + 6*(a^10*x^12 + a^8*x^10)*(a^2*x^2 + 1)^(5/2) + 15*(a^11*x^13
 + 2*a^9*x^11 + a^7*x^9)*(a^2*x^2 + 1)^2 + 20*(a^12*x^14 + 3*a^10*x^12 + 3*
a^8*x^10 + a^6*x^8)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*x^15 + 4*a^11*x^13 + 6*a
^9*x^11 + 4*a^7*x^9 + a^5*x^7)*(a^2*x^2 + 1) + 6*(a^14*x^16 + 5*a^12*x^14 +
 10*a^10*x^12 + 10*a^8*x^10 + 5*a^6*x^8 + a^4*x^6)*sqrt(a^2*x^2 + 1))*log(a
*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2 \operatorname{asinh}(a\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*asinh(a*x)^4),x)
```

```
[Out] int(1/(x^2*asinh(a*x)^4), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{asinh}^4(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)**4,x)
```

```
[Out] Integral(1/(x**2*asinh(a*x)**4), x)
```

3.74 $\int x^4\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=182

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}-\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{64a^5}+\frac{\sqrt{\frac{\pi}{5}}\,\mathrm{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{320a^5}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+$$

```
[Out] 1/1600*erf(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-1/1600*erfi(5^(
1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-1/192*erf(3^(1/2)*arcsinh(a*x
)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+1/192*erfi(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2
)*Pi^(1/2)/a^5+1/32*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5-1/32*erfi(arcsinh(
a*x)^(1/2))*Pi^(1/2)/a^5+1/5*x^5*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}-\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{64a^5}+\frac{\sqrt{\frac{\pi}{5}}\,\mathrm{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{320a^5}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x^5*Sqrt[ArcSinh[a*x]])/5 + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(32*a^5) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(64*a^5) + (Sqrt[Pi/5]*Erf[Sqr
t[5]*Sqrt[ArcSinh[a*x]]])/(320*a^5) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(
32*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(64*a^5) - (Sqrt[Pi
/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(320*a^5)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt{\sinh^{-1}(ax)}\,dx &= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} - \frac{1}{10}a\int \frac{x^5}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx \\
&= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^5(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{10a^5} \\
&= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} + \frac{i\,\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8\sqrt{x}} - \frac{5i\sinh(3x)}{16\sqrt{x}} + \frac{i\sinh(5x)}{16\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{10a^5} \\
&= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{160a^5} + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \sinh^{-1}\right.}{32a^5} \\
&= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{320a^5} - \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{320a^5} - \\
&= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{160a^5} - \frac{\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\sinh^{-1}(\right.}{160a^5} \\
&= \frac{1}{5}x^5\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{64a^5} + \frac{\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\right.}{3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 161, normalized size = 0.88

$$\frac{\frac{\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-5\sinh^{-1}(ax)\right)}{160\sqrt{5}\sqrt{-\sinh^{-1}(ax)}} - \frac{\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-3\sinh^{-1}(ax)\right)}{32\sqrt{3}\sqrt{-\sinh^{-1}(ax)}} + \frac{\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-\sinh^{-1}(ax)\right)}{16\sqrt{-\sinh^{-1}(ax)}} - \frac{1}{16}\Gamma\left(\frac{3}{2},\sinh^{-1}(ax)\right) + \frac{\Gamma\left(\frac{3}{2},3\sinh\right.}{32}}{a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[3/2, -5*ArcSinh[a*x]])/(160*Sqrt[5]*Sqrt[-ArcSin
h[a*x]]) - (Sqrt[ArcSinh[a*x]]*Gamma[3/2, -3*ArcSinh[a*x]])/(32*Sqrt[3]*Sqr
t[-ArcSinh[a*x]]) + (Sqrt[ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/(16*Sqrt
[-ArcSinh[a*x]]) - Gamma[3/2, ArcSinh[a*x]]/16 + Gamma[3/2, 3*ArcSinh[a*x]]
/(32*Sqrt[3]) - Gamma[3/2, 5*ArcSinh[a*x]]/(160*Sqrt[5]))/a^5
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{\operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*sqrt(arcsinh(a*x)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{\operatorname{arcsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{\operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \sqrt{\operatorname{asinh}(a\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asinh(a*x)^(1/2),x)
```

```
[Out] int(x^4*asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{\operatorname{asinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(asinh(a*x)), x)
```

3.75 $\int x^3\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-$$

```
[Out] 1/64*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+1/64*erfi(2^(1/2)
*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/256*erf(2*arcsinh(a*x)^(1/2))*P
i^(1/2)/a^4-1/256*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4-3/32*arcsinh(a*x)
^(1/2)/a^4+1/4*x^4*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (-3*Sqrt[ArcSinh[a*x]])/(32*a^4) + (x^4*Sqrt[ArcSinh[a*x]])/4 - (Sqrt[Pi]*E
rf[2*Sqrt[ArcSinh[a*x]]])/(256*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[
a*x]]])/(32*a^4) - (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a^4) + (Sqrt[
Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(32*a^4)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{\sinh^{-1}(ax)}\,dx &= \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{1}{8}a\int \frac{x^4}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx\\
&= \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^4}\\
&= \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{8\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{8a^4}\\
&= -\frac{3\sqrt{\sinh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{64a^4} + \text{Subst}(\ldots\\
&= -\frac{3\sqrt{\sinh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{128a^4} - \text{Subst}\left(\int \ldots\right.\\
&= -\frac{3\sqrt{\sinh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{64a^4} - \text{Subst}(\ldots\\
&= -\frac{3\sqrt{\sinh^{-1}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\sinh^{-1}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sin\ldots\right.}{32a^4}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 0.73

$$\frac{\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2}, -4\sinh^{-1}(ax)\right) - 4\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2}, -2\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(\Gamma\left(\frac{3}{2}, 4\sinh^{-1}(\ldots\right.\right.}{128a^4\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]] - 4*Sqrt[2]*Sqrt[ArcSinh[a*
x]]*Gamma[3/2, -2*ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(-4*Sqrt[2]*Gamma[3/2
, 2*ArcSinh[a*x]] + Gamma[3/2, 4*ArcSinh[a*x]]))/(128*a^4*Sqrt[-ArcSinh[a*x
]])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*sqrt(arcsinh(a*x)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{\operatorname{asinh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x)^(1/2),x)
```

```
[Out] int(x^3*asinh(a*x)^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(asinh(a*x)), x)
```

3.76 $\int x^2\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=120

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3}$$

```
[Out] 1/144*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-1/144*erfi(3^(1/
2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-1/16*erf(arcsinh(a*x)^(1/2))*Pi
^(1/2)/a^3+1/16*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3+1/3*x^3*arcsinh(a*x)^
(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x^3*Sqrt[ArcSinh[a*x]])/3 - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a^3) +
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(48*a^3) + (Sqrt[Pi]*Erfi[Sqrt
[ArcSinh[a*x]]])/(16*a^3) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(
48*a^3)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^2\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{1}{6}a\int \frac{x^3}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{6a^3}$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{i\,\text{Subst}\left(\int\left(\frac{3i\sinh(x)}{4\sqrt{x}} - \frac{i\sinh(3x)}{4\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{6a^3}$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{24a^3} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(\ldots\right)}{8a^3}$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{48a^3} - \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{48a^3} - \ldots$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{24a^3} - \frac{\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\sinh^{-1}(\ldots}\right)}{24a^3}$$

$$= \frac{1}{3}x^3\sqrt{\sinh^{-1}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^3} + \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{48a^3} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\ldots}\right)}{16\ldots}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 0.84

$$\frac{\sqrt{3}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2}, -3\sinh^{-1}(ax)\right) - 9\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2}, -\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(9\Gamma\left(\frac{3}{2}, \sinh^{-1}(ax)\right)\ldots\right.}{72a^3\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, -3*ArcSinh[a*x]] - 9*Sqrt[ArcSinh[a*
x]]*Gamma[3/2, -ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(9*Gamma[3/2, ArcSinh[a
*x]] - Sqrt[3]*Gamma[3/2, 3*ArcSinh[a*x]]))/(72*a^3*Sqrt[-ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(arcsinh(a*x)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{\mathrm{asinh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x)^(1/2),x)
```

```
[Out] int(x^2*asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(asinh(a*x)), x)
```

3.77 $\int x\sqrt{\sinh^{-1}(ax)}\, dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}+\frac{\sqrt{\sinh^{-1}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)}$$

```
[Out] -1/32*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-1/32*erfi(2^(1/2
)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+1/4*arcsinh(a*x)^(1/2)/a^2+1/2*x
^2*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2}+\frac{\sqrt{\sinh^{-1}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Sqrt[ArcSinh[a*x]]/(4*a^2) + (x^2*Sqrt[ArcSinh[a*x]])/2 - (Sqrt[Pi/2]*Erf[S
qrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a^2) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSin
h[a*x]]])/(16*a^2)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{\sinh^{-1}(ax)}\,dx &= \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{1}{4}a\int \frac{x^2}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx\\
&= \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^2}\\
&= \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{4a^2}\\
&= \frac{\sqrt{\sinh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^2}\\
&= \frac{\sqrt{\sinh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a^2} - \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,d\right.}{1}\\
&= \frac{\sqrt{\sinh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a^2} - \frac{\text{Subst}\left(\int e\right.}{}\\
&= \frac{\sqrt{\sinh^{-1}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\sinh^{-1}(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a^2} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh\right.}{16a^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.56

$$\frac{\frac{\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \Gamma\left(\frac{3}{2},2\sinh^{-1}(ax)\right)}{8\sqrt{2}\,a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + Gam
ma[3/2, 2*ArcSinh[a*x]])/(8*Sqrt[2]*a^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="giac")

[Out] integrate(x*sqrt(arcsinh(a*x)), x)

maple [A] time = 0.30, size = 75, normalized size = 0.81

$$\frac{\sqrt{2}\,\left(8\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,x^{2}a^{2}+4\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}-\pi\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\right)-\pi\operatorname{erfi}\left(\sqrt{2}\,\sqrt{\operatorname{arcsir}}\right.\right.}{32\sqrt{\pi}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arcsinh(a*x)^(1/2),x)

[Out] 1/32*2^(1/2)*(8*2^(1/2)*arcsinh(a*x)^(1/2)*Pi^(1/2)*x^2*a^2+4*2^(1/2)*arcsi
nh(a*x)^(1/2)*Pi^(1/2)-Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))-Pi*erfi(2^(1/2)*a
rcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x*sqrt(arcsinh(a*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{\operatorname{asinh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*asinh(a*x)^(1/2),x)

[Out] int(x*asinh(a*x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*asinh(a*x)**(1/2),x)

[Out] Integral(x*sqrt(asinh(a*x)), x)

3.78 $\int \sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}-\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}+x\sqrt{\sinh^{-1}(ax)}$$

```
[Out] 1/4*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a-1/4*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2
)/a+x*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}-\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}+x\sqrt{\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcSinh[a*x]],x]
```

```
[Out] x*Sqrt[ArcSinh[a*x]] + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(4*a) - (Sqrt[Pi]
*Erfi[Sqrt[ArcSinh[a*x]]])/(4*a)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sinh^{-1}(ax)}\,dx &= x\sqrt{\sinh^{-1}(ax)} - \frac{1}{2}a\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx\\
&= x\sqrt{\sinh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a}\\
&= x\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a}\\
&= x\sqrt{\sinh^{-1}(ax)} + \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a} - \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a}\\
&= x\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a} - \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.85

$$-\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \Gamma\left(\frac{3}{2},\sinh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -1/2*((Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] +
Gamma[3/2, ArcSinh[a*x]])/a
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x)), x)
```

maple [A] time = 0.30, size = 42, normalized size = 0.79

$$\frac{4\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,xa+\pi\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-\pi\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4\sqrt{\pi}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2),x)
```

```
[Out] 1/4*(4*arcsinh(a*x)^(1/2)*Pi^(1/2)*x*a+Pi*erf(arcsinh(a*x)^(1/2))-Pi*erfi(a
rcsinh(a*x)^(1/2)))/Pi^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{\operatorname{asinh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2),x)
```

```
[Out] int(asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(asinh(a*x)), x)
```

3.79 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{x} dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{\sqrt{\sinh^{-1}(ax)}}{x},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^(1/2)/x,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[a*x]]/x,x]
[Out] Defer[Int][Sqrt[ArcSinh[a*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{x} dx = \int \frac{\sqrt{\sinh^{-1}(ax)}}{x} dx$$

Mathematica [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/x,x]
[Out] Integrate[Sqrt[ArcSinh[a*x]]/x, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="giac")
```

[Out] integrate(sqrt(arcsinh(a*x))/x, x)

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/x,x)
[Out] int(arcsinh(a*x)^(1/2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(a*x))/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sqrt{\operatorname{asinh}(a\,x)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(1/2)/x,x)
[Out] int(asinh(a*x)^(1/2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/x,x)
[Out] Integral(sqrt(asinh(a*x))/x, x)
```

3.80 $\int x^4 \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=330

$$\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^5}-\frac{3\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{3200a^5}-\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{200a^5}+\frac{3\sqrt{\frac{\pi}{5}}\,\mathrm{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{3200a^5}$$

```
[Out] 1/5*x^5*arcsinh(a*x)^(3/2)+3/16000*erf(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*
Pi^(1/2)/a^5+3/16000*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-
1/384*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-1/384*erfi(3^(1/
2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+3/64*erf(arcsinh(a*x)^(1/2))*Pi
^(1/2)/a^5+3/64*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5-4/25*(a^2*x^2+1)^(1/2
)*arcsinh(a*x)^(1/2)/a^5+2/25*x^2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a^3-
3/50*x^4*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] time = 0.71, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5717, 5657, 3307, 2180, 2204, 2205, 5669, 5448}

$$\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^5}-\frac{3\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{3200a^5}-\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{200a^5}+\frac{3\sqrt{\frac{\pi}{5}}\,\mathrm{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{3200a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(25*a^5) + (2*x^2*Sqrt[1 + a^2*x^
2]*Sqrt[ArcSinh[a*x]])/(25*a^3) - (3*x^4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x
]])/(50*a) + (x^5*ArcSinh[a*x]^(3/2))/5 + (3*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]
]])/(64*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(200*a^5) - (3*
Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi/5]*Erf[
Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x
]]])/(64*a^5) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(200*a^5) - (
3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi/5]*E
rfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(3200*a^5)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{5}x^5\sinh^{-1}(ax)^{3/2} - \frac{1}{10}(3a)\int \frac{x^5\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5\sinh^{-1}(ax)^{3/2} + \frac{3}{100}\int \frac{x^4}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{6\int \frac{x^3\sqrt{}}{\sqrt{}}}{}$$

$$= \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a} + \frac{1}{5}x^5\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Sub}}{}$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a}$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a}$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a}$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a}$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a}$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{50a}$$

Mathematica [A] time = 0.13, size = 152, normalized size = 0.46

$$\frac{\frac{9\sqrt{5}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-5\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{125\sqrt{3}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-3\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \frac{2250\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} - 2250\Gamma\left(\frac{5}{2},\sin\right.}{36000a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((9*Sqrt[5]*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -5*ArcSinh[a*x]])/Sqrt[ArcSinh[a
*x]] + (125*Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -3*ArcSinh[a*x]])/Sqrt[-A
rcSinh[a*x]] + (2250*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[Ar
cSinh[a*x]] - 2250*Gamma[5/2, ArcSinh[a*x]] + 125*Sqrt[3]*Gamma[5/2, 3*ArcS
inh[a*x]] - 9*Sqrt[5]*Gamma[5/2, 5*ArcSinh[a*x]])/(36000*a^5)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^4*arcsinh(a*x)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arcsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^(3/2),x)
[Out] int(x^4*arcsinh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4*arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\mathrm{asinh}\left(a\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asinh(a*x)^(3/2),x)
[Out] int(x^4*asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \operatorname{asinh}^{\frac{3}{2}}{\left(ax \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**(3/2),x)
[Out] Integral(x**4*asinh(a*x)**(3/2), x)
```

3.81 $\int x^3 \sinh^{-1}(ax)^{3/2}\, dx$

Optimal. Leaf size=199

$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a^4}+\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{128a^4}+\frac{3\sqrt{\pi}\,\mathrm{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a^4}-\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(a\right.}}{128a^4}$$

```
[Out] -3/32*arcsinh(a*x)^(3/2)/a^4+1/4*x^4*arcsinh(a*x)^(3/2)+3/256*erf(2^(1/2)*a
rcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-3/256*erfi(2^(1/2)*arcsinh(a*x)^(1/
2))*2^(1/2)*Pi^(1/2)/a^4-3/2048*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4+3/20
48*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4+9/64*x*(a^2*x^2+1)^(1/2)*arcsinh
(a*x)^(1/2)/a^3-3/32*x^3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] time = 0.49, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a^4}+\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{128a^4}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a^4}-\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\right.}}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (9*x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(64*a^3) - (3*x^3*Sqrt[1 + a^2*x
^2]*Sqrt[ArcSinh[a*x]])/(32*a) - (3*ArcSinh[a*x]^(3/2))/(32*a^4) + (x^4*Arc
Sinh[a*x]^(3/2))/4 - (3*Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(2048*a^4) + (3
*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(128*a^4) + (3*Sqrt[Pi]*Erfi[2
*Sqrt[ArcSinh[a*x]]])/(2048*a^4) - (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[
a*x]]])/(128*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{4}x^4 \sinh^{-1}(ax)^{3/2} - \frac{1}{8}(3a)\int \frac{x^4\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} + \frac{3}{64}\int \frac{x^3}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{9\int \frac{x^2\sqrt{\mathrm{si}}}{\sqrt{1}}}{3}$$
$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} + \frac{1}{4}x^4\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subs}}{}$$
$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\,\mathrm{sinh}$$
$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\,\mathrm{sinh}$$
$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\,\mathrm{sinh}$$
$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\,\mathrm{sinh}$$
$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\,\mathrm{sinh}$$
$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{32a} - \frac{3\sinh^{-1}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\,\mathrm{sinh}$$

Mathematica [**A**] time = 0.04, size = 102, normalized size = 0.51

$$\frac{-\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-4\sinh^{-1}(ax)\right) + 8\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-2\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(\Gamma\left(\frac{5}{2},4\sinh^{-1}(a\right.\right.}{512a^4\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-(Sqrt[ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]]) + 8*Sqrt[2]*Sqrt[ArcSinh
[a*x]]*Gamma[5/2, -2*ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(-8*Sqrt[2]*Gamma[
5/2, 2*ArcSinh[a*x]] + Gamma[5/2, 4*ArcSinh[a*x]]))/(512*a^4*Sqrt[-ArcSinh[
a*x]])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple **[F(-2)]** time = 180.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arcsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(3/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asinh}(a\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x)^(3/2),x)
```

```
[Out] int(x^3*asinh(a*x)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3*asinh(a*x)**(3/2), x)
```

3.82 $\int x^2 \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=179

$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{96a^3}-\frac{3\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{96a^3}$$

```
[Out] 1/3*x^3*arcsinh(a*x)^(3/2)+1/288*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi
^(1/2)/a^3+1/288*erfi(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-3/32
*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3-3/32*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2
)/a^3+1/3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a^3-1/6*x^2*(a^2*x^2+1)^(1/2
)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] time = 0.37, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5717, 5657, 3307, 2180, 2204, 2205, 5669, 5448}

$$-\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{96a^3}-\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{96a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(3*a^3) - (x^2*Sqrt[1 + a^2*x^2]*Sqr
t[ArcSinh[a*x]])/(6*a) + (x^3*ArcSinh[a*x]^(3/2))/3 - (3*Sqrt[Pi]*Erf[Sqrt[
ArcSinh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(96
*a^3) - (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erfi[S
qrt[3]*Sqrt[ArcSinh[a*x]]])/(96*a^3)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{3}x^3 \sinh^{-1}(ax)^{3/2} - \frac{1}{2}a\int \frac{x^3\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} + \frac{1}{12}\int \frac{x^2}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{\int \frac{x\sqrt{\sinh^{-1}}}{\sqrt{1+a^2x}}}{3a}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{}\right.}{1}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{6a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{3/2} - \frac{3\sqrt{\pi}\,\text{erf}\left(\right.}{3}$$

Mathematica [A] time = 0.04, size = 102, normalized size = 0.57

$$\frac{-\sqrt{3}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-3\sinh^{-1}(ax)\right)+27\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-\sinh^{-1}(ax)\right)+\sqrt{-\sinh^{-1}(ax)}\left(27\Gamma\left(\frac{5}{2},\sinh^{-1}(a\right.\right.}{216a^3\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-(Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -3*ArcSinh[a*x]]) + 27*Sqrt[ArcSin
h[a*x]]*Gamma[5/2, -ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(27*Gamma[5/2, ArcS
inh[a*x]] - Sqrt[3]*Gamma[5/2, 3*ArcSinh[a*x]]))/(216*a^3*Sqrt[-ArcSinh[a*x
]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcsinh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^(3/2),x)
[Out] int(x^2*arcsinh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arsinh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2*arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asinh}(a\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x)^(3/2),x)
[Out] int(x^2*asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**(3/2),x)
[Out] Integral(x**2*asinh(a*x)**(3/2), x)
```

3.83 $\int x \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=122

$$-\frac{3\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2}-\frac{3x\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{8a}+\frac{\sinh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\,\mathrm{si}$$

```
[Out] 1/4*arcsinh(a*x)^(3/2)/a^2+1/2*x^2*arcsinh(a*x)^(3/2)-3/128*erf(2^(1/2)*arc
sinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+3/128*erfi(2^(1/2)*arcsinh(a*x)^(1/2)
)*2^(1/2)*Pi^(1/2)/a^2-3/8*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] time = 0.22, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5663, 5758, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a^2}-\frac{3x\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{8a}+\frac{\sinh^{-1}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(8*a) + ArcSinh[a*x]^(3/2)/(4*a
^2) + (x^2*ArcSinh[a*x]^(3/2))/2 - (3*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(64*a^2) + (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x \sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} - \frac{1}{4}(3a)\int \frac{x^2\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} + \frac{3}{16}\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{3\int \frac{\sqrt{\sinh^{-1}}}{\sqrt{1+a^2}}}{8a}$$

$$= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sin}{\sqrt{x}}\right.}{16}$$

$$= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,d\right.}{16a^2}$$

$$= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,d\right.}{32a^2}$$

$$= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} - \frac{3\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x\right.}{64a^2}$$

$$= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} - \frac{3\,\mathrm{Subst}\left(\int e^{-2x^2}\,dx,\right.}{32a^2}$$

$$= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{8a} + \frac{\sinh^{-1}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{3/2} - \frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh\right.}}{64a^2}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.43

$$\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \Gamma\left(\frac{5}{2},2\sinh^{-1}(ax)\right)}{16\sqrt{2}\,a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Gam
ma[5/2, 2*ArcSinh[a*x]])/(16*Sqrt[2]*a^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{arsinh}\,(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^(3/2), x)
```

maple [A] time = 0.35, size = 102, normalized size = 0.84

$$-\frac{\sqrt{2}\,\left(-32\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,x^{2}a^{2}+24\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}\,xa-16\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\right.}{128\sqrt{\pi}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^(3/2),x)
```

```
[Out] -1/128*2^(1/2)*(-32*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+24*2^(1/2)*
arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a-16*arcsinh(a*x)^(3/2)*2^(
1/2)*Pi^(1/2)+3*Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))-3*Pi*erfi(2^(1/2)*arcsin
h(a*x)^(1/2)))/Pi^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\operatorname{asinh}(a\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x)^(3/2),x)
```

```
[Out] int(x*asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x*asinh(a*x)**(3/2), x)
```

3.84 $\int \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=81

$$-\frac{3\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{2a}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}+\frac{3\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}+x\sinh^{-1}(ax)^{3/2}$$

```
[Out] x*arcsinh(a*x)^(3/2)+3/8*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+3/8*erfi(arcsin
h(a*x)^(1/2))*Pi^(1/2)/a-3/2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5653, 5717, 5657, 3307, 2180, 2204, 2205}

$$-\frac{3\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{2a}+\frac{3\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}+\frac{3\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a}+x\sinh^{-1}(ax)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(2*a) + x*ArcSinh[a*x]^(3/2) + (3
*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(8*a) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*
x]]])/(8*a)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sinh^{-1}(ax)^{3/2}\,dx = x\sinh^{-1}(ax)^{3/2} - \frac{1}{2}(3a)\int \frac{x\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$
$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3}{4}\int \frac{1}{\sqrt{\sinh^{-1}(ax)}}\,dx$$
$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a}$$
$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a} + \frac{3\,\mathrm{Su}}{}$$
$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a} +$$
$$= -\frac{3\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{2a} + x\sinh^{-1}(ax)^{3/2} + \frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\right.}{8}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.58

$$\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{5}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} - \Gamma\left(\frac{5}{2},\sinh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] - Gamma
[5/2, ArcSinh[a*x]])/(2*a)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(3/2), x)
```

maple [A] time = 0.34, size = 65, normalized size = 0.80

$$\frac{8 \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{\pi}\, xa - 12\sqrt{\operatorname{arcsinh}(ax)}\, \sqrt{\pi}\, \sqrt{a^2x^2+1} + 3\pi \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right) + 3\pi \operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8\sqrt{\pi}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2),x)
```

```
[Out] 1/8*(8*arcsinh(a*x)^(3/2)*Pi^(1/2)*x*a-12*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*
x^2+1)^(1/2)+3*Pi*erf(arcsinh(a*x)^(1/2))+3*Pi*erfi(arcsinh(a*x)^(1/2)))/Pi
^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a\,x)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2),x)
```

```
[Out] int(asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}^{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**(3/2), x)
```

3.85 $\int \frac{\sinh^{-1}(ax)^{3/2}}{x} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\sinh^{-1}(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^(3/2)/x,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(3/2)/x,x]
[Out] Defer[Int][ArcSinh[a*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{x} \, dx = \int \frac{\sinh^{-1}(ax)^{3/2}}{x} \, dx$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(3/2)/x,x]
[Out] Integrate[ArcSinh[a*x]^(3/2)/x, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}(ax)^{\frac{3}{2}}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(3/2)/x, x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)/x,x)
[Out] int(arcsinh(a*x)^(3/2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(3/2)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{asinh}(a\,x)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(3/2)/x,x)
[Out] int(asinh(a*x)^(3/2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)/x,x)
[Out] Integral(asinh(a*x)**(3/2)/x, x)
```

3.86 $\int x^4 \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=379

$$\frac{15\sqrt{\pi}\operatorname{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{128a^5}-\frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{1280a^5}-\frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{240a^5}+\frac{3\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(\right.}{6400a^5}$$

```
[Out] 1/5*x^5*arcsinh(a*x)^(5/2)+3/32000*erf(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*
Pi^(1/2)/a^5-3/32000*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-
5/2304*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+5/2304*erfi(3^(
1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+15/128*erf(arcsinh(a*x)^(1/2)
)*Pi^(1/2)/a^5-15/128*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5-4/15*arcsinh(a*
x)^(3/2)*(a^2*x^2+1)^(1/2)/a^5+2/15*x^2*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2
)/a^3-1/10*x^4*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a+2/5*x*arcsinh(a*x)^(1
/2)/a^4-1/15*x^3*arcsinh(a*x)^(1/2)/a^2+3/100*x^5*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 1.00, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$\frac{15\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{128a^5}-\frac{\sqrt{3\pi}\operatorname{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{1280a^5}-\frac{\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{240a^5}+\frac{3\sqrt{\frac{\pi}{5}}\operatorname{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}\right.}{6400a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (2*x*Sqrt[ArcSinh[a*x]])/(5*a^4) - (x^3*Sqrt[ArcSinh[a*x]])/(15*a^2) + (3*x
^5*Sqrt[ArcSinh[a*x]])/100 - (4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(15*a
^5) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(15*a^3) - (x^4*Sqrt[1 +
 a^2*x^2]*ArcSinh[a*x]^(3/2))/(10*a) + (x^5*ArcSinh[a*x]^(5/2))/5 + (15*Sqr
t[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(128*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[Arc
Sinh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(1280
*a^5) + (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(6400*a^5) - (15*Sqr
t[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(128*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[A
rcSinh[a*x]]])/(240*a^5) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(1
280*a^5) - (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(6400*a^5)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{5}x^5 \sinh^{-1}(ax)^{5/2} - \frac{1}{2}a\int \frac{x^5 \sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}x^5\sinh^{-1}(ax)^{5/2} + \frac{3}{20}\int x^4\sqrt{\sinh^{-1}(ax)}\,dx + \frac{2\int \frac{x}{}}{}$$

$$= \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} + \frac{2x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{10a} + \frac{1}{5}$$

$$= -\frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+a^2x}}{1}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3}}{15a^5}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3}}{15a^5}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3}}{15a^5}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3}}{15a^5}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3}}{15a^5}$$

$$= \frac{2x\sqrt{\sinh^{-1}(ax)}}{5a^4} - \frac{x^3\sqrt{\sinh^{-1}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\sinh^{-1}(ax)} - \frac{4\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3}}{15a^5}$$

Mathematica [A] time = 0.13, size = 152, normalized size = 0.40

$$\frac{\frac{27\sqrt{5}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-5\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \frac{625\sqrt{3}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-3\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{33750\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} - 33750\Gamma\left(\frac{7}{2}\right.}{540000a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((27*Sqrt[5]*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -5*ArcSinh[a*x]])/Sqrt[-ArcSinh[
a*x]] + (625*Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -3*ArcSinh[a*x]])/Sqrt[
ArcSinh[a*x]] + (33750*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[-
ArcSinh[a*x]] - 33750*Gamma[7/2, ArcSinh[a*x]] + 625*Sqrt[3]*Gamma[7/2, 3*A
rcSinh[a*x]] - 27*Sqrt[5]*Gamma[7/2, 5*ArcSinh[a*x]])/(540000*a^5)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming a near 0Simplification assuming a
 near 0Simplification assuming a near 0Simplification assuming a near 0Simp
lification assuming a near 0Simplification assuming t_nostep near 0Simplifi
cation assuming a near 0Simplification assuming a near 0Simplification assu
ming a near 0Simplification assuming a near 0Simplification assuming a near
 0Simplification assuming t_nostep near 0Simplification assuming a near 0Si
mplification assuming a near 0Simplification assuming a near 0Simplificatio
n assuming a near 0Simplification assuming a near 0Simplification assuming
t_nostep near 0Simplification assuming a near 0Simplification assuming a ne
ar 0Simplification assuming a near 0Simplification assuming a near 0Simplif
ication assuming a near 0Simplification assuming t_nostep near 0sym2poly/r2
sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arcsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\mathrm{asinh}\left(a\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asinh(a*x)^(5/2),x)
```

```
[Out] int(x^4*asinh(a*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

3.87 $\int x^3 \sinh^{-1}(ax)^{5/2}\, dx$

Optimal. Leaf size=247

$$-\frac{15\sqrt{\pi}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{512a^4}-\frac{15\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{s}\right.}{512a^4}$$

```
[Out] -3/32*arcsinh(a*x)^(5/2)/a^4+1/4*x^4*arcsinh(a*x)^(5/2)+15/1024*erf(2^(1/2)
*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+15/1024*erfi(2^(1/2)*arcsinh(a*x)
^(1/2))*2^(1/2)*Pi^(1/2)/a^4-15/16384*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^
4-15/16384*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4+15/64*x*arcsinh(a*x)^(3/
2)*(a^2*x^2+1)^(1/2)/a^3-5/32*x^3*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a-22
5/2048*arcsinh(a*x)^(1/2)/a^4-45/256*x^2*arcsinh(a*x)^(1/2)/a^2+15/256*x^4*
arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{512a^4}-\frac{15\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\right.}{512a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-225*Sqrt[ArcSinh[a*x]])/(2048*a^4) - (45*x^2*Sqrt[ArcSinh[a*x]])/(256*a^2
) + (15*x^4*Sqrt[ArcSinh[a*x]])/256 + (15*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
(3/2))/(64*a^3) - (5*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(32*a) - (3*
ArcSinh[a*x]^(5/2))/(32*a^4) + (x^4*ArcSinh[a*x]^(5/2))/4 - (15*Sqrt[Pi]*Er
f[2*Sqrt[ArcSinh[a*x]]])/(16384*a^4) + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcS
inh[a*x]]])/(512*a^4) - (15*Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(16384*a^4
) + (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(512*a^4)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{4}x^4 \sinh^{-1}(ax)^{5/2} - \frac{1}{8}(5a)\int \frac{x^4 \sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{5x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{32a} + \frac{1}{4}x^4 \sinh^{-1}(ax)^{5/2} + \frac{15}{64}\int x^3\sqrt{\sinh^{-1}(ax)}\,dx + \frac{15\int}{}$$

$$= \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{32a} +$$

$$= -\frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-}}{}$$

$$= -\frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-}}{}$$

$$= -\frac{45\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\,\sinh}{64a^3}$$

$$= -\frac{225\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\,\sinh}{64a^3}$$

$$= -\frac{225\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\,\sinh}{64a^3}$$

$$= -\frac{225\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\,\sinh}{64a^3}$$

$$= -\frac{225\sqrt{\sinh^{-1}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\sinh^{-1}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\sinh^{-1}(ax)} + \frac{15x\sqrt{1+a^2x^2}\,\sinh}{64a^3}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 0.41

$$\frac{\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-4\sinh^{-1}(ax)\right) - 16\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-2\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(\Gamma\left(\frac{7}{2},4\sinh^{-1}\right.\right.}{2048a^4\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]] - 16*Sqrt[2]*Sqrt[ArcSinh[a
*x]]*Gamma[7/2, -2*ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(-16*Sqrt[2]*Gamma[7
/2, 2*ArcSinh[a*x]] + Gamma[7/2, 4*ArcSinh[a*x]]))/(2048*a^4*Sqrt[-ArcSinh[
a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arcsinh}(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arsinh}(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\operatorname{asinh}(a\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x)^(5/2),x)
```

```
[Out] int(x^3*asinh(a*x)^(5/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asinh}^{\frac{5}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**3*asinh(a*x)**(5/2), x)
```

3.88 $\int x^2 \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=210

$$-\frac{15\sqrt{\pi}\operatorname{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^3}+\frac{5\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{576a^3}+\frac{15\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^3}-\frac{5\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}\right.}}{576a^3}$$

```
[Out] 1/3*x^3*arcsinh(a*x)^(5/2)+5/1728*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*P
i^(1/2)/a^3-5/1728*erfi(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-15
/64*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3+15/64*erfi(arcsinh(a*x)^(1/2))*Pi^
(1/2)/a^3+5/9*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a^3-5/18*x^2*arcsinh(a*x
)^(3/2)*(a^2*x^2+1)^(1/2)/a-5/6*x*arcsinh(a*x)^(1/2)/a^2+5/36*x^3*arcsinh(a
*x)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}\operatorname{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^3}+\frac{5\sqrt{\frac{\pi}{3}}\operatorname{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{576a^3}+\frac{15\sqrt{\pi}\operatorname{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{64a^3}-\frac{5\sqrt{\frac{\pi}{3}}\operatorname{Erfi}\left(\sqrt{3}\sqrt{\sinh\right.}}{576a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-5*x*Sqrt[ArcSinh[a*x]])/(6*a^2) + (5*x^3*Sqrt[ArcSinh[a*x]])/36 + (5*Sqrt
[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x]^(3/2))/(18*a) + (x^3*ArcSinh[a*x]^(5/2))/3 - (15*Sqrt[Pi]*Erf[Sqrt[A
rcSinh[a*x]]])/(64*a^3) + (5*Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(5
76*a^3) + (15*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(64*a^3) - (5*Sqrt[Pi/3]*E
rfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(576*a^3)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^2 \sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{3}x^3\sinh^{-1}(ax)^{5/2} - \frac{1}{6}(5a)\int \frac{x^3\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{5x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\sinh^{-1}(ax)^{5/2} + \frac{5}{12}\int x^2\sqrt{\sinh^{-1}(ax)}\,dx + \frac{5\int}{}$$

$$= \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{18a} + \frac{1}{3}x^3$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}}{18}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}}{18}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}}{18}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}}{18}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}}{18}$$

$$= -\frac{5x\sqrt{\sinh^{-1}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\sinh^{-1}(ax)} + \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}}{18}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 0.48

$$\frac{\sqrt{3}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-3\sinh^{-1}(ax)\right) - 81\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\left(81\Gamma\left(\frac{7}{2},\sinh^{-1}\right.\right.}{648a^3\sqrt{-\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -3*ArcSinh[a*x]] - 81*Sqrt[ArcSinh[a
*x]]*Gamma[7/2, -ArcSinh[a*x]] + Sqrt[-ArcSinh[a*x]]*(81*Gamma[7/2, ArcSinh
[a*x]] - Sqrt[3]*Gamma[7/2, 3*ArcSinh[a*x]]))/(648*a^3*Sqrt[-ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming a near 0Simplification assuming a
 near 0Simplification assuming a near 0Simplification assuming a near 0Simp
lification assuming a near 0Simplification assuming t_nostep near 0Simplifi
cation assuming a near 0Simplification assuming a near 0Simplification assu
ming a near 0Simplification assuming a near 0Simplification assuming a near
 0Simplification assuming t_nostep near 0Simplification assuming a near 0Si
mplification assuming a near 0Simplification assuming a near 0Simplificatio
n assuming a near 0Simplification assuming a near 0Simplification assuming
t_nostep near 0Simplification assuming a near 0Simplification assuming a ne
ar 0Simplification assuming a near 0Simplification assuming a near 0Simplif
ication assuming a near 0Simplification assuming t_nostep near 0sym2poly/r2
sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{asinh}(a\,x)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x)^(5/2),x)
```

```
[Out] int(x^2*asinh(a*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asinh}^{\frac{5}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**2*asinh(a*x)**(5/2), x)
```

3.89 $\int x \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=152

$$-\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a^2}-\frac{5x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{8a}+\frac{\sinh^{-1}(ax)^{5/2}}{4a^2}+$$

```
[Out] 1/4*arcsinh(a*x)^(5/2)/a^2+1/2*x^2*arcsinh(a*x)^(5/2)-15/512*erf(2^(1/2)*ar
csinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-15/512*erfi(2^(1/2)*arcsinh(a*x)^(1/
2))*2^(1/2)*Pi^(1/2)/a^2-5/8*x*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a+15/64
*arcsinh(a*x)^(1/2)/a^2+15/32*x^2*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a^2}-\frac{5x\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{8a}+\frac{\sinh^{-1}(ax)^{5/2}}{4a^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (15*Sqrt[ArcSinh[a*x]])/(64*a^2) + (15*x^2*Sqrt[ArcSinh[a*x]])/32 - (5*x*Sq
rt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(8*a) + ArcSinh[a*x]^(5/2)/(4*a^2) + (x
^2*ArcSinh[a*x]^(5/2))/2 - (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/
(256*a^2) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a^2)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int x\sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2} - \frac{1}{4}(5a)\int \frac{x^2\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{5x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{8a} + \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2} + \frac{15}{16}\int x\sqrt{\sinh^{-1}(ax)}\,dx + \frac{5\int \frac{\sinh}{\sqrt{}}}{}$$

$$= \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2}$$

$$= \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2}$$

$$= \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\sinh^{-1}(ax)^{5/2}$$

$$= \frac{15\sqrt{\sinh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} +$$

$$= \frac{15\sqrt{\sinh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} +$$

$$= \frac{15\sqrt{\sinh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} +$$

$$= \frac{15\sqrt{\sinh^{-1}(ax)}}{64a^2} + \frac{15}{32}x^2\sqrt{\sinh^{-1}(ax)} - \frac{5x\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{8a} + \frac{\sinh^{-1}(ax)^{5/2}}{4a^2} +$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.34

$$\frac{\frac{\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \Gamma\left(\frac{7}{2},2\sinh^{-1}(ax)\right)}{32\sqrt{2}\,a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[7/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + Gam
ma[7/2, 2*ArcSinh[a*x]])/(32*Sqrt[2]*a^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming a near 0Simplification assuming a
 near 0Simplification assuming a near 0Simplification assuming a near 0Simp
lification assuming a near 0Simplification assuming t_nostep near 0Simplifi
cation assuming a near 0Simplification assuming a near 0Simplification assu
ming a near 0Simplification assuming a near 0Simplification assuming a near
 0Simplification assuming t_nostep near 0Simplification assuming a near 0Si
mplification assuming a near 0Simplification assuming a near 0Simplificatio
n assuming a near 0Simplification assuming a near 0Simplification assuming
t_nostep near 0Simplification assuming a near 0Simplification assuming a ne
ar 0Simplification assuming a near 0Simplification assuming a near 0Simplif
ication assuming a near 0Simplification assuming t_nostep near 0sym2poly/r2
sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
Value
```

maple [A] time = 0.35, size = 136, normalized size = 0.89

$$-\frac{\sqrt{2}\,\left(-128\operatorname{arcsinh}(ax)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,x^2a^2+160\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,xa-120\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^(5/2),x)
```

```
[Out] -1/512*2^(1/2)*(-128*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+160*arcsin
h(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a-120*2^(1/2)*arcsinh(a*x
)^(1/2)*Pi^(1/2)*x^2*a^2-64*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)-60*2^(1/2)*
arcsinh(a*x)^(1/2)*Pi^(1/2)+15*Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))+15*Pi*erf
i(2^(1/2)*arcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arsinh}(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\mathrm{asinh}\left(a\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x)^(5/2),x)
```

```
[Out] int(x*asinh(a*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asinh}^{\frac{5}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x*asinh(a*x)**(5/2), x)
```

3.90 $\int \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=94

$$-\frac{5\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{2a}+\frac{15\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a}-\frac{15\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a}+x\sinh^{-1}(ax)^{5/2}+\frac{15}{4}x\sqrt{\mathrm{si}}$$

```
[Out] x*arcsinh(a*x)^(5/2)+15/16*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a-15/16*erfi(ar
csinh(a*x)^(1/2))*Pi^(1/2)/a-5/2*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a+15/
4*x*arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5653, 5717, 5779, 3308, 2180, 2204, 2205}

$$-\frac{5\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{2a}+\frac{15\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a}-\frac{15\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a}+x\sinh^{-1}(ax)^{5/2}+\frac{15}{4}x\sqrt{\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(5/2),x]
```

```
[Out] (15*x*Sqrt[ArcSinh[a*x]])/4 - (5*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(2*a
) + x*ArcSinh[a*x]^(5/2) + (15*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a) - (
15*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(16*a)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}(ax)^{5/2}\,dx &= x\sinh^{-1}(ax)^{5/2} - \frac{1}{2}(5a)\int \frac{x\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} + \frac{15}{4}\int \sqrt{\sinh^{-1}(ax)}\,dx \\
&= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} - \frac{1}{8}(15a)\int \frac{x}{\sqrt{1+a^2x^2}\,\sqrt{s}} \\
&= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} - \frac{15\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x\right.}{8a} \\
&= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} + \frac{15\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{si}\right.}{16a} \\
&= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} + \frac{15\,\text{Subst}\left(\int e^{-x^2}\,dx, x, \right.}{8a} \\
&= \frac{15}{4}x\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}{2a} + x\sinh^{-1}(ax)^{5/2} + \frac{15\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right.}{16a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.48

$$-\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{7}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \Gamma\left(\frac{7}{2},\sinh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(5/2),x]
```

```
[Out] -1/2*((Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] +
Gamma[7/2, ArcSinh[a*x]])/a
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming a near 0Simplification assuming a
 near 0Simplification assuming a near 0Simplification assuming a near 0Simp
lification assuming a near 0Simplification assuming t_nostep near 0Simplifi
cation assuming a near 0Simplification assuming a near 0Simplification assu
ming a near 0Simplification assuming a near 0Simplification assuming a near
 0Simplification assuming t_nostep near 0Simplification assuming a near 0Si
mplification assuming a near 0Simplification assuming a near 0Simplificatio
n assuming a near 0Simplification assuming a near 0Simplification assuming
t_nostep near 0Simplification assuming a near 0Simplification assuming a ne
ar 0Simplification assuming a near 0Simplification assuming a near 0Simplif
ication assuming a near 0Simplification assuming t_nostep near 0sym2poly/r2
sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
Value
```

maple [A] time = 0.34, size = 78, normalized size = 0.83

$$\frac{16 \operatorname{arcsinh}(ax)^{\frac{5}{2}} \sqrt{\pi}\, xa - 40 \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{\pi}\, \sqrt{a^2x^2+1} + 60\sqrt{\operatorname{arcsinh}(ax)}\, \sqrt{\pi}\, xa + 15\pi \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(a\right.}}{16\sqrt{\pi}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2),x)
```

```
[Out] 1/16*(16*arcsinh(a*x)^(5/2)*Pi^(1/2)*x*a-40*arcsinh(a*x)^(3/2)*Pi^(1/2)*(a^
2*x^2+1)^(1/2)+60*arcsinh(a*x)^(1/2)*Pi^(1/2)*x*a+15*Pi*erf(arcsinh(a*x)^(1
/2))-15*Pi*erfi(arcsinh(a*x)^(1/2)))/Pi^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a\,x)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2),x)
```

```
[Out] int(asinh(a*x)^(5/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}^{\frac{5}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**(5/2), x)
```

3.91 $\int \frac{\sinh^{-1}(ax)^{5/2}}{x} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\sinh^{-1}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^(5/2)/x,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(5/2)/x,x]
[Out] Defer[Int][ArcSinh[a*x]^(5/2)/x, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{x} dx = \int \frac{\sinh^{-1}(ax)^{5/2}}{x} dx$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(5/2)/x,x]
[Out] Integrate[ArcSinh[a*x]^(5/2)/x, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(5/2)/x, x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)/x,x)
[Out] int(arcsinh(a*x)^(5/2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(5/2)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{asinh}(a\,x)^{5/2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^(5/2)/x,x)
[Out] int(asinh(a*x)^(5/2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{\frac{5}{2}}(ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)/x,x)
[Out] Integral(asinh(a*x)**(5/2)/x, x)
```

3.92 $\int \frac{x^4}{\sqrt{\sinh^{-1}(ax)}} \, dx$

Optimal. Leaf size=163

$$\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-$$

```
[Out] 1/160*erf(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+1/160*erfi(5^(1/
2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+1/16*erf(arcsinh(a*x)^(1/2))*Pi
^(1/2)/a^5+1/16*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5-1/32*erf(3^(1/2)*arcs
inh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-1/32*erfi(3^(1/2)*arcsinh(a*x)^(1/2))*
3^(1/2)*Pi^(1/2)/a^5
```

Rubi [A] time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5669, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{3\pi}\,\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{\pi}{5}}\,\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(32
*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erfi[Sqr
t[3]*Sqrt[ArcSinh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[
a*x]]])/(32*a^5)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{\sinh^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{x}} - \frac{3\cosh(3x)}{16\sqrt{x}} + \frac{\cosh(5x)}{16\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} - \frac{3\,\text{Subst}\left(\int \frac{\cos
\end{aligned}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{si}}{16a^5}$$

$$= \frac{\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{16a^5} + \frac{\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{16a^5} + \frac{\text{Subst}\left(\int e^{-x^2}\right.}{}$$

$$= \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{32a^5} + \frac{\sqrt{}}{}$$

Mathematica [A] time = 0.11, size = 151, normalized size = 0.93

$$\frac{\frac{\sqrt{5}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-5\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{5\sqrt{3}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} + \frac{10\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} - 10\Gamma\left(\frac{1}{2},\sinh^{-1}(ax\right.}{160a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[5]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -5*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x
]] + (5*Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]])/Sqrt[-ArcSi
nh[a*x]] + (10*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[
a*x]] - 10*Gamma[1/2, ArcSinh[a*x]] + 5*Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]]
- Sqrt[5]*Gamma[1/2, 5*ArcSinh[a*x]])/(160*a^5)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(arcsinh(a*x)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asinh(a*x)^(1/2),x)
```

```
[Out] int(x^4/asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(asinh(a*x)), x)
```

3.93 $\int \frac{x^3}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4}$$

```
[Out] 1/16*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/16*erfi(2^(1/2)
*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/32*erf(2*arcsinh(a*x)^(1/2))*Pi
^(1/2)/a^4+1/32*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4
```

Rubi [A] time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5669, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(32*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[a*x]]])/(8*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(32*a^4)
 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(8*a^4)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx\right.}{8a}$$

$$= -\frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a^4} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a^4} -$$

Mathematica [A] time = 0.08, size = 99, normalized size = 0.91

$$\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} + \frac{2\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}} - 2\sqrt{2}\,\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right) + \Gamma\left(\frac{1}{2},4\sinh^{-1}(ax)\right)}{32a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + (2*
Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]]
 - 2*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] + Gamma[1/2, 4*ArcSinh[a*x]])/(32*a
^4)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(arcsinh(a*x)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asinh(a*x)^(1/2),x)
```

```
[Out] int(x^3/asinh(a*x)^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(asinh(a*x)), x)
```

3.94 $\int \frac{x^2}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}-\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}$$

```
[Out] 1/24*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+1/24*erfi(3^(1/2)
*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-1/8*erf(arcsinh(a*x)^(1/2))*Pi^(1
/2)/a^3-1/8*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5669, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}-\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(8*a^3) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/(8*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(8*a^3) + (Sq
rt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(8*a^3)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}}+\frac{\cosh(3x)}{4\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^3}-\frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^3}-\frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{si}\right.}{8a^3}$$

$$= \frac{\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\text{Subst}\left(\int e^{x^2}\,\right.}{}$$

$$= -\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}-\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{e}}{}$$

Mathematica [A] time = 0.08, size = 99, normalized size = 0.94

$$\frac{\frac{\sqrt{3}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}}+\frac{3\sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-\sinh^{-1}(ax)\right)}{\sqrt{-\sinh^{-1}(ax)}}+3\Gamma\left(\frac{1}{2},\sinh^{-1}(ax)\right)-\sqrt{3}\,\Gamma\left(\frac{1}{2},3\sinh^{-1}(ax)\right)}{24a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x
]] + (3*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] +
 3*Gamma[1/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]])/(24*a^3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\text{arsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(arcsinh(a*x)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{arsinh}(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\operatorname{asinh}(a\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a*x)^(1/2),x)
```

```
[Out] int(x^2/asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{asinh}(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(asinh(a*x)), x)
```

3.95 $\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}$$

```
[Out] -1/8*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+1/8*erfi(2^(1/2)*
arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}-\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -(Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a^2) + (Sqrt[Pi/2]*Erfi[Sq
rt[2]*Sqrt[ArcSinh[a*x]]])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^2}+\frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^2}\\
&= -\frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a^2}+\frac{\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a^2}\\
&= -\frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}+\frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a^2}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.83

$$
\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}}+\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)}{4\sqrt{2}\,a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Gam
ma[1/2, 2*ArcSinh[a*x]])/(4*Sqrt[2]*a^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x}{\sqrt{\text{arsinh}\,(ax)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(arcsinh(a*x)), x)
```

maple [A] time = 0.21, size = 37, normalized size = 0.59

$$-\frac{\sqrt{\pi}\,\sqrt{2}\,\left(\mathrm{erf}\left(\sqrt{2}\,\sqrt{\mathrm{arcsinh}\,(ax)}\right)-\mathrm{erfi}\left(\sqrt{2}\,\sqrt{\mathrm{arcsinh}\,(ax)}\right)\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^(1/2),x)
```

```
[Out] -1/8*Pi^(1/2)*2^(1/2)*(erf(2^(1/2)*arcsinh(a*x)^(1/2))-erfi(2^(1/2)*arcsinh
(a*x)^(1/2)))/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\mathrm{arsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{\mathrm{asinh}\,(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a*x)^(1/2),x)
```

```
[Out] int(x/asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\mathrm{asinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(asinh(a*x)), x)
```

3.96 $\int \frac{1}{\sqrt{\sinh^{-1}(ax)}} \, dx$

Optimal. Leaf size=43

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}$$

```
[Out] 1/2*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+1/2*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2
)/a
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5657, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(2*a) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]
]])/(2*a)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a}$$
$$= \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a} + \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a}$$
$$= \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{2a}$$

Mathematica [**A**] time = 0.03, size = 47, normalized size = 1.09

$$\frac{\frac{\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-\sinh^{-1}(ax)\right)}{\sqrt{\sinh^{-1}(ax)}} - \Gamma\left(\frac{1}{2},\sinh^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[ArcSinh[a*x]],x]
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] - Gamma
[1/2, ArcSinh[a*x]])/(2*a)
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(arcsinh(a*x)), x)
```

maple [**A**] time = 0.19, size = 24, normalized size = 0.56

$$\frac{\sqrt{\pi}\,\left(\text{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right) + \text{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(1/2),x)
```

```
[Out] 1/2*Pi^(1/2)*(erf(arcsinh(a*x)^(1/2))+erfi(arcsinh(a*x)^(1/2)))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(arcsinh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a*x)^(1/2),x)
[Out] int(1/asinh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(1/2),x)
[Out] Integral(1/sqrt(asinh(a*x)), x)
```

3.97 $\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x\sqrt{\sinh^{-1}(ax)}},x\right)$$

[Out] Unintegrable(1/x/arcsinh(a*x)^(1/2),x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*Sqrt[ArcSinh[a*x]]),x]

[Out] Defer[Int][1/(x*Sqrt[ArcSinh[a*x]]), x]

Rubi steps

$$\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx=\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*Sqrt[ArcSinh[a*x]]),x]

[Out] Integrate[1/(x*Sqrt[ArcSinh[a*x]]), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="giac")

```
[Out] integrate(1/(x*sqrt(arcsinh(a*x))), x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^(1/2),x)
[Out] int(1/x/arcsinh(a*x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(arcsinh(a*x))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\,\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)^(1/2)),x)
[Out] int(1/(x*asinh(a*x)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(asinh(a*x))), x)
```

3.98 $\int \frac{1}{x^2 \sqrt{\sinh^{-1}(ax)}} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x^2 \sqrt{\sinh^{-1}(ax)}}, x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)^(1/2),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \sqrt{\sinh^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[ArcSinh[a*x]]),x]
[Out] Defer[Int][1/(x^2*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \sqrt{\sinh^{-1}(ax)}} \, dx = \int \frac{1}{x^2 \sqrt{\sinh^{-1}(ax)}} \, dx$$

Mathematica [A] time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\sinh^{-1}(ax)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/(x^2*Sqrt[ArcSinh[a*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\operatorname{arsinh}(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*sqrt(arcsinh(a*x))), x)
```

maple [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{\operatorname{arcsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(1/x^2/arcsinh(a*x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt(arcsinh(a*x))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x^2\,\sqrt{\mathrm{asinh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*asinh(a*x)^(1/2)),x)
```

```
[Out] int(1/(x^2*asinh(a*x)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{\operatorname{asinh}{\left(ax \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(asinh(a*x))), x)
```

3.99 $\int \frac{x^4}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{5\pi}\,\mathrm{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}-$$

```
[Out] -1/8*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5+1/8*erfi(arcsinh(a*x)^(1/2))*Pi^(
1/2)/a^5+3/16*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-3/16*erf
i(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-1/16*erf(5^(1/2)*arcsinh
(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+1/16*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(
1/2)*Pi^(1/2)/a^5-2*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5665, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}+\frac{3\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}-\frac{\sqrt{5\pi}\,\mathrm{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/(8*a^5) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(16*a^
5) - (Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(16*a^5) + (Sqrt[Pi]*Erfi
[Sqrt[ArcSinh[a*x]]])/(8*a^5) - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x
]]])/(16*a^5) + (Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(16*a^5)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int\left(\frac{\sinh(x)}{8\sqrt{x}} - \frac{9\sinh(3x)}{16\sqrt{x}} + \frac{5\sinh(5x)}{16\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\mathrm{Subst}\left(\int\frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a^5} + \frac{5\,\mathrm{Subst}\left(\int\frac{\sinh(5x)}{\sqrt{x}}\,dx, x, \sinh^{-1}\right.}{8a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} + \frac{\mathrm{Subst}\left(\int\frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} -$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{4a^5} + \frac{\mathrm{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(a\right.}{4a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{16a^5} - \frac{\sqrt{5\pi}\,\mathrm{erf}\left(\right.}{}$$

Mathematica [A] time = 0.31, size = 216, normalized size = 1.15

$$\frac{-e^{-5\sinh^{-1}(ax)} + 3e^{-3\sinh^{-1}(ax)} - 2e^{-\sinh^{-1}(ax)} - 2e^{\sinh^{-1}(ax)} + 3e^{3\sinh^{-1}(ax)} - e^{5\sinh^{-1}(ax)} + \sqrt{5}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-E^(-5*ArcSinh[a*x]) + 3/E^(3*ArcSinh[a*x]) - 2/E^ArcSinh[a*x] - 2*E^ArcSi
nh[a*x] + 3*E^(3*ArcSinh[a*x]) - E^(5*ArcSinh[a*x]) + Sqrt[5]*Sqrt[-ArcSinh
[a*x]]*Gamma[1/2, -5*ArcSinh[a*x]] - 3*Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/
2, -3*ArcSinh[a*x]] + 2*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]] + 2*S
qrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh[a*x]] - 3*Sqrt[3]*Sqrt[ArcSinh[a*x]]*G
amma[1/2, 3*ArcSinh[a*x]] + Sqrt[5]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 5*ArcSinh
[a*x]])/(16*a^5*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^4/arcsinh(a*x)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^(3/2),x)
[Out] int(x^4/arcsinh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{asinh}(a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asinh(a*x)^(3/2),x)
[Out] int(x^4/asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**(3/2),x)
[Out] Integral(x**4/asinh(a*x)**(3/2), x)
```

3.100 $\int \frac{x^3}{\sinh^{-1}(ax)^{3/2}} dx$

Optimal. Leaf size=138

$$\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4}+\frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4}$$

```
[Out] -1/4*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/4*erfi(2^(1/2)*
arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+1/4*erf(2*arcsinh(a*x)^(1/2))*Pi^(
1/2)/a^4+1/4*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4-2*x^3*(a^2*x^2+1)^(1/2
)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4}-\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[2*Sqrt[Ar
cSinh[a*x]]])/(4*a^4) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(2*a^4
) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(4*a^4) - (Sqrt[Pi/2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(2*a^4)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{2\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^4} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(a\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a^4} - \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a^4} - \frac{\text{S}}{}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a^4} - \frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(a}\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\right.}{4a}$$

Mathematica [A] time = 0.04, size = 126, normalized size = 0.91

$$\frac{2\sinh\left(2\sinh^{-1}(ax)\right) - \sinh\left(4\sinh^{-1}(ax)\right) + \sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, -4\sinh^{-1}(ax)\right) - \sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, -2\right.}{4a^4\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - Sqrt[2]*Sqrt[-ArcSinh[a*
x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*A
rcSinh[a*x]] - Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]] + 2*Sinh[2*Arc
Sinh[a*x]] - Sinh[4*ArcSinh[a*x]])/(4*a^4*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{asinh}(a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asinh(a*x)^(3/2),x)
```

```
[Out] int(x^3/asinh(a*x)^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/asinh(a*x)**(3/2), x)
```

3.101 $\int \frac{x^2}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\,\mathrm{erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{2}{a}$$

```
[Out] 1/4*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3-1/4*erfi(arcsinh(a*x)^(1/2))*Pi^(1
/2)/a^3-1/4*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+1/4*erfi(3
^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-2*x^2*(a^2*x^2+1)^(1/2)/a/a
rcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5665, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/(4*a^3) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(4*a^3)
- (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(4*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sq
rt[ArcSinh[a*x]]])/(4*a^3)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
```

```
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4\sqrt{x}}+\frac{3\sinh(3x)}{4\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}dx, x, \sinh^{-1}(ax)\right)}{2a^3} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}dx, x, \sinh^{-}\right.}{2a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}dx, x, \sinh^{-1}(ax)\right)}{4a^3} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}dx, x, \sinh^{-1}(ax)\right)}{4a^3} -$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\text{Subst}\left(\int e^{-x^2}dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a^3} - \frac{\text{Subst}\left(\int e^{x^2}dx, x, \sqrt{\sinh^{-1}(a\right.}}{2a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{4a^3} - \frac{\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{4a^3} - \frac{\sqrt{\pi}\,\text{erfi}\left(\right.}{}$$

Mathematica [A] time = 0.14, size = 140, normalized size = 1.08

$$\frac{-e^{-3\sinh^{-1}(ax)}+e^{-\sinh^{-1}(ax)}+e^{\sinh^{-1}(ax)}-e^{3\sinh^{-1}(ax)}+\sqrt{3}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)-\sqrt{-\sinh^{-1}(a}}{4a^3\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-E^(-3*ArcSinh[a*x]) + E^(-ArcSinh[a*x]) + E^ArcSinh[a*x] - E^(3*ArcSinh[a
*x]) + Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]] - Sqrt[-ArcS
inh[a*x]]*Gamma[1/2, -ArcSinh[a*x]] - Sqrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh
[a*x]] + Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 3*ArcSinh[a*x]])/(4*a^3*Sqrt
[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{asinh}(a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a*x)^(3/2),x)
```

```
[Out] int(x^2/asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/asinh(a*x)**(3/2), x)
```

3.102 $\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] 1/2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+1/2*erfi(2^(1/2)*a
rcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-2*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x
)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}-\frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi/2]*Erf[Sqrt[2]*S
qrt[ArcSinh[a*x]]])/a^2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/a^2
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
```

```
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx &= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}+\frac{\mathrm{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{2\,\mathrm{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a^2}+\frac{2\,\mathrm{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a^2}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a^2}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.93

$$
-\frac{\sinh\left(2\sinh^{-1}(ax)\right)}{a^2\sqrt{\sinh^{-1}(ax)}}+\frac{\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)}{\sqrt{2}\,a^2\sqrt{\sinh^{-1}(ax)}}-\frac{\Gamma\left(\frac{1}{2},2\sinh^{-1}(ax)\right)}{\sqrt{2}\,a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/(Sqrt[2]*a^2*Sqrt[ArcSinh
[a*x]]) - Gamma[1/2, 2*ArcSinh[a*x]]/(Sqrt[2]*a^2) - Sinh[2*ArcSinh[a*x]]/(
a^2*Sqrt[ArcSinh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^(3/2), x)
```

maple [A] time = 0.36, size = 80, normalized size = 0.95

$$\frac{\sqrt{2}\,\left(-2\sqrt{2}\,\sqrt{\operatorname{arcsinh}\left(ax\right)}\,\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}\,xa+\pi\operatorname{arcsinh}\left(ax\right)\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}\left(ax\right)}\right)+\pi\operatorname{arcsinh}\left(ax\right)\operatorname{erfi}\left(\right.}{2\sqrt{\pi}\,a^{2}\operatorname{arcsinh}\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^(3/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*2^(1/2)*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a+P
i*arcsinh(a*x)*erf(2^(1/2)*arcsinh(a*x)^(1/2))+Pi*arcsinh(a*x)*erfi(2^(1/2)
*arcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2/arcsinh(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a*x)^(3/2),x)
```

```
[Out] int(x/asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x/asinh(a*x)**(3/2), x)
```

3.103 $\int \frac{1}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}$$

```
[Out] -erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a-2*(
a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\sinh^{-1}(ax)}}-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[
a*x]]])/a + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/a
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sinh^{-1}(ax)^{3/2}}\,dx &= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + (2a)\int \frac{x}{\sqrt{1+a^2x^2}\,\sqrt{\sinh^{-1}(ax)}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a} + \frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{2\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a} + \frac{2\,\mathrm{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a} + \frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 1.08

$$\frac{-e^{-\sinh^{-1}(ax)} - e^{\sinh^{-1}(ax)} + \sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, -\sinh^{-1}(ax)\right) + \sqrt{\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2}, \sinh^{-1}(ax)\right)}{a\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(-3/2),x]
```

```
[Out] (-E^(-ArcSinh[a*x]) - E^ArcSinh[a*x] + Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcS
inh[a*x]] + Sqrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh[a*x]])/(a*Sqrt[ArcSinh[a*
x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-3/2), x)
```

maple [A] time = 0.36, size = 65, normalized size = 1.02

$$-\frac{\operatorname{arcsinh}(ax)\,\pi\,\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-\operatorname{arcsinh}(ax)\,\pi\,\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)+2\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}}{\sqrt{\pi}\,a\operatorname{arcsinh}(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(3/2),x)
```

```
[Out] -(arcsinh(a*x)*Pi*erf(arcsinh(a*x)^(1/2))-arcsinh(a*x)*Pi*erfi(arcsinh(a*x)
^(1/2))+2*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2))/Pi^(1/2)/a/arcsinh
(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{asinh}(a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a*x)^(3/2),x)
```

```
[Out] int(1/asinh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**(-3/2), x)
```

3.104 $\int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sinh^{-1}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^(3/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^(3/2)),x]
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx = \int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx$$

Mathematica [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh^{-1}(ax)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^(3/2)),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^(3/2)), x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^(3/2),x)
[Out] int(1/x/arcsinh(a*x)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(a*x)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asinh}(a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)^(3/2)),x)
[Out] int(1/(x*asinh(a*x)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**(3/2),x)
[Out] Integral(1/(x*asinh(a*x)**(3/2)), x)
```

3.105 $\int \frac{x^4}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}-\frac{3\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}+\frac{5\sqrt{5\pi}\,\mathrm{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{24a^5}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}$$

```
[Out] 1/12*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5+1/12*erfi(arcsinh(a*x)^(1/2))*Pi^
(1/2)/a^5-3/8*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-3/8*erfi
(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+5/24*erf(5^(1/2)*arcsinh(
a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+5/24*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1
/2)*Pi^(1/2)/a^5-2/3*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)-16/3*x^3/a^
2/arcsinh(a*x)^(1/2)-20/3*x^5/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}-\frac{3\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{8a^5}+\frac{5\sqrt{5\pi}\,\mathrm{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{24a^5}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (16*x^3)/(3*a^2*Sqrt[
ArcSinh[a*x]]) - (20*x^5)/(3*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcSi
nh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(8*a^5
) + (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(24*a^5) + (Sqrt[Pi]*Erf
i[Sqrt[ArcSinh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a
*x]]])/(8*a^5) + (5*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(24*a^5)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{8\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(10a)\int \frac{x^5}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx \cdots$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{100}{3}\int \frac{x^4}{\sqrt{\sinh^{-1}(ax)}}\,dx + \cdots$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx \cdots\right.}{a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}} + \frac{\cosh \cdots}{4\sqrt{\cdots}}\right.\right.}{a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}\,dx, x, \text{si}\cdots\right.}{12a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \sinh^{\cdots}\right.}{24a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{25\,\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\text{si}\cdots}\right.}{12a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{20x^5}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{12a^5} - \frac{3\cdots}{\cdots}$$

Mathematica [A] time = 0.36, size = 343, normalized size = 1.54

$$\frac{-\frac{e^{5\sinh^{-1}(ax)}\left(10\sinh^{-1}(ax)+1\right)+10\sqrt{5}\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-5\sinh^{-1}(ax)\right)}{48\sinh^{-1}(ax)^{3/2}} + \frac{e^{3\sinh^{-1}(ax)}\left(6\sinh^{-1}(ax)+1\right)+6\sqrt{3}\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-3\sinh\cdots\right.}{16\sinh^{-1}(ax)^{3/2}} \cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-1/48*(E^(5*ArcSinh[a*x])*(1 + 10*ArcSinh[a*x]) + 10*Sqrt[5]*(-ArcSinh[a*x
])^(3/2)*Gamma[1/2, -5*ArcSinh[a*x]])/ArcSinh[a*x]^(3/2) + (E^(3*ArcSinh[a*
x])*(1 + 6*ArcSinh[a*x]) + 6*Sqrt[3]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -3*Ar
cSinh[a*x]])/(16*ArcSinh[a*x]^(3/2)) - (E^ArcSinh[a*x]*(1 + 2*ArcSinh[a*x])
 + 2*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]])/(24*ArcSinh[a*x]^(3/2
)) - (1 - 2*ArcSinh[a*x] + 2*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)*Gamma[1/2, A
rcSinh[a*x]])/(24*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)) + (1 - 6*ArcSinh[a*x]
+ 6*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 3*ArcSinh[a*x]
])/(16*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)) - (1 - 10*ArcSinh[a*x] + 10*S
qrt[5]*E^(5*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 5*ArcSinh[a*x]])/(4
8*E^(5*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)))/a^5
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\operatorname{asinh}\left(a\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asinh(a*x)^(5/2),x)
```

```
[Out] int(x^4/asinh(a*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**4/asinh(a*x)**(5/2), x)
```

3.106 $\int \frac{x^3}{\sinh^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=167

$$-\frac{2\sqrt{\pi}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}+\frac{\sqrt{2\pi}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}+\frac{2\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}-\frac{\sqrt{2\pi}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\right.}}{3a^4}$$

```
[Out] -2/3*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4+2/3*erfi(2*arcsinh(a*x)^(1/2))*
Pi^(1/2)/a^4+1/3*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/3*e
rfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-2/3*x^3*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)^(3/2)-4*x^2/a^2/arcsinh(a*x)^(1/2)-16/3*x^4/arcsinh(a*x)
^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5667, 5774, 5669, 5448, 3308, 2180, 2204, 2205, 12}

$$-\frac{2\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}+\frac{\sqrt{2\pi}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}+\frac{2\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4}-\frac{\sqrt{2\pi}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh\right.}}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (4*x^2)/(a^2*Sqrt[Arc
Sinh[a*x]]) - (16*x^4)/(3*Sqrt[ArcSinh[a*x]]) - (2*Sqrt[Pi]*Erf[2*Sqrt[ArcS
inh[a*x]]])/(3*a^4) + (Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a^4)
+ (2*Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(3*a^4) - (Sqrt[2*Pi]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(3*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{2\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(8a)\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} + \frac{64}{3}\int \frac{x^3}{\sqrt{\sinh^{-1}(ax)}}\,dx + \frac{8\int}{}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x,\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \sinh\right.}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \sinh\right.}{3a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(\right.}{3a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh}\right.}{3a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\sinh^{-1}(ax)}} - \frac{16x^4}{3\sqrt{\sinh^{-1}(ax)}} - \frac{2\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a^4} + \cdots$$

Mathematica [A] time = 0.43, size = 174, normalized size = 1.04

$$\frac{2\sinh\left(2\sinh^{-1}(ax)\right) - \sinh\left(4\sinh^{-1}(ax)\right) + 4\sinh^{-1}(ax)\left(e^{-2\sinh^{-1}(ax)} + e^{2\sinh^{-1}(ax)} - \sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (4*ArcSinh[a*x]*(E^(-2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x]) - Sqrt[2]*Sqrt[-A
rcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma
[1/2, 2*ArcSinh[a*x]]) - 4*ArcSinh[a*x]*(E^(-4*ArcSinh[a*x]) + E^(4*ArcSinh
[a*x]) - 2*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - 2*Sqrt[ArcSinh
[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]]) + 2*Sinh[2*ArcSinh[a*x]] - Sinh[4*ArcSin
h[a*x]])/(12*a^4*ArcSinh[a*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asinh(a*x)^(5/2),x)
```

```
[Out] int(x^3/asinh(a*x)^(5/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**3/asinh(a*x)**(5/2), x)
```

3.107 $\int \frac{x^2}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{2a^3}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\,\mathrm{erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{2a^3}$$

```
[Out] -1/6*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3-1/6*erfi(arcsinh(a*x)^(1/2))*Pi^(
1/2)/a^3+1/2*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+1/2*erfi(
3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-2/3*x^2*(a^2*x^2+1)^(1/2)/
a/arcsinh(a*x)^(3/2)-8/3*x/a^2/arcsinh(a*x)^(1/2)-4*x^3/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205, 5657}

$$-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{2a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3}+\frac{\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x)/(3*a^2*Sqrt[Arc
Sinh[a*x]]) - (4*x^3)/Sqrt[ArcSinh[a*x]] - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]
])/(6*a^3) + (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(2*a^3) - (Sqrt[P
i]*Erfi[Sqrt[ArcSinh[a*x]]])/(6*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSin
h[a*x]]])/(2*a^3)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{4\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}dx}{3a} + (2a)\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}dx$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + 12\int \frac{x^2}{\sqrt{\sinh^{-1}(ax)}}dx + \frac{8\int}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}dx, x, \sinh^{-}\right.}{3a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{4\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}dx, x, \sinh^{-1}(a\right.}{3a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\text{Subst}\left(\int e^{-x^2}dx, x, \sqrt{\sinh^{-}}\right.}{3a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{4\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a^3} + \frac{4}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} + \frac{4\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a^3} + \frac{4}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{4x^3}{\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{6a^3} + \frac{\sqrt{3}}{}$$

Mathematica [A] time = 0.16, size = 225, normalized size = 1.40

$$-\frac{\frac{e^{3\sinh^{-1}(ax)}\left(6\sinh^{-1}(ax)+1\right)+6\sqrt{3}\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-3\sinh^{-1}(ax)\right)}{12\sinh^{-1}(ax)^{3/2}} + \frac{e^{\sinh^{-1}(ax)}\left(2\sinh^{-1}(ax)+1\right)+2\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-\sinh^{-1}(ax)\right)}{12\sinh^{-1}(ax)^{3/2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-1/12*(E^(3*ArcSinh[a*x])*(1 + 6*ArcSinh[a*x]) + 6*Sqrt[3]*(-ArcSinh[a*x])
^(3/2)*Gamma[1/2, -3*ArcSinh[a*x]])/ArcSinh[a*x]^(3/2) + (E^ArcSinh[a*x]*(1
 + 2*ArcSinh[a*x]) + 2*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]])/(12
*ArcSinh[a*x]^(3/2)) + (1 - 2*ArcSinh[a*x] + 2*E^ArcSinh[a*x]*ArcSinh[a*x]^
(3/2)*Gamma[1/2, ArcSinh[a*x]])/(12*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)) - (1
 - 6*ArcSinh[a*x] + 6*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1
/2, 3*ArcSinh[a*x]])/(12*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)))/a^3
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/arcsinh(a*x)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^(5/2),x)
[Out] int(x^2/arcsinh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a*x)^(5/2),x)
[Out] int(x^2/asinh(a*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**(5/2),x)
[Out] Integral(x**2/asinh(a*x)**(5/2), x)
```

3.108 $\int \frac{x}{\sinh^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=118

$$-\frac{2\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}+\frac{2\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}-\frac{2x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8x}{3\sqrt{\sinh}}$$

```
[Out] -2/3*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+2/3*erfi(2^(1/2)*
arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-2/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(
a*x)^(3/2)-4/3/a^2/arcsinh(a*x)^(1/2)-8/3*x^2/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5667, 5774, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5675}

$$-\frac{2\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}+\frac{2\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}-\frac{2x\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}}-\frac{8}{3\sqrt{\sinh}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcSinh[a
*x]]) - (8*x^2)/(3*Sqrt[ArcSinh[a*x]]) - (2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[Arc
Sinh[a*x]]])/(3*a^2) + (2*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{2\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(4a)\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}} + \frac{16}{3}\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}} + \frac{16\,\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx,x,\sin$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx,x,\sinh$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}} - \frac{4\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\sinh^{-1}$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}} - \frac{8\,\text{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\sinh$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\sinh^{-1}(ax)}} - \frac{8x^2}{3\sqrt{\sinh^{-1}(ax)}} - \frac{2\sqrt{2\pi}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a^2}$$

Mathematica [A] time = 0.18, size = 98, normalized size = 0.83

$$-\frac{\sinh\left(2\sinh^{-1}(ax)\right) + 2\sinh^{-1}(ax)\left(e^{-2\sinh^{-1}(ax)} + e^{2\sinh^{-1}(ax)} - \sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\sinh^{-1}(ax)\right) - \sqrt{}\right.}{3a^2\sinh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcSinh[a*x]^(5/2),x]
```

```
[Out] -1/3*(2*ArcSinh[a*x]*(E^(-2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x]) - Sqrt[2]*Sq
rt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*
Gamma[1/2, 2*ArcSinh[a*x]]) + Sinh[2*ArcSinh[a*x]])/(a^2*ArcSinh[a*x]^(3/2)
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^(5/2), x)
```

maple [A] time = 0.33, size = 119, normalized size = 1.01

$$-\frac{\sqrt{2}\,\left(4\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,x^2a^2+\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,xa+2\operatorname{arcsinh}(ax)^2\pi\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsin}}\right.\right.}{3\sqrt{\pi}\,a^2\operatorname{arcsinh}(ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^(5/2),x)
```

```
[Out] -1/3*2^(1/2)*(4*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2+2^(1/2)*arcsinh
(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a+2*arcsinh(a*x)^2*Pi*erf(2^(1/2)*
arcsinh(a*x)^(1/2))-2*arcsinh(a*x)^2*Pi*erfi(2^(1/2)*arcsinh(a*x)^(1/2))+2*
arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arcsinh(a*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a*x)^(5/2),x)
```

```
[Out] int(x/asinh(a*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x/asinh(a*x)**(5/2), x)
```

3.109 $\int \frac{1}{\sinh^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=84

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^{3/2}}+\frac{2\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}-\frac{4x}{3\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] 2/3*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+2/3*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2
)/a-2/3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)-4/3*x/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5655, 5774, 5657, 3307, 2180, 2204, 2205}

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sinh^{-1}(ax)^{3/2}}+\frac{2\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a}-\frac{4x}{3\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (4*x)/(3*Sqrt[ArcSinh[a*x
]]) + (2*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(3*a) + (2*Sqrt[Pi]*Erfi[Sqrt[Ar
cSinh[a*x]]])/(3*a)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sinh^{-1}(ax)^{5/2}}\,dx &= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{1}{3}(2a)\int \frac{x}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{3/2}}\,dx \\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{4}{3}\int \frac{1}{\sqrt{\sinh^{-1}(ax)}}\,dx \\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{4\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a} \\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a} + \frac{2\,\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\right.}{} \\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{4\,\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{3a} + \frac{4\,\text{Subst}\left(\int \right.}{} \\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{4x}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{3a} + \frac{2\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right.}{3a}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 105, normalized size = 1.25

$$-\frac{e^{-\sinh^{-1}(ax)}\left(e^{2\sinh^{-1}(ax)} + 2e^{2\sinh^{-1}(ax)}\sinh^{-1}(ax) - 2\sinh^{-1}(ax) + 2e^{\sinh^{-1}(ax)}\left(-\sinh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2}, -\sinh^{-1}(a\right.\right.}{3a\sinh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(-5/2),x]
```

```
[Out] -1/3*(1 + E^(2*ArcSinh[a*x]) - 2*ArcSinh[a*x] + 2*E^(2*ArcSinh[a*x])*ArcSin
h[a*x] + 2*E^ArcSinh[a*x]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]] +
 2*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)*Gamma[1/2, ArcSinh[a*x]])/(a*E^ArcSinh
[a*x]*ArcSinh[a*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-5/2), x)
```

maple [A] time = 0.32, size = 81, normalized size = 0.96

$$\frac{-\frac{4 \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{\pi}\, xa}{3}+\frac{2 \operatorname{arcsinh}(ax)^{2} \pi \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3}+\frac{2 \operatorname{arcsinh}(ax)^{2} \pi \operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3}-\frac{2 \sqrt{\operatorname{arcsinh}(ax)}\, \sqrt{\pi}\, \sqrt{a^{2}x^{2}+1}}{3}}{\sqrt{\pi}\, a \operatorname{arcsinh}(ax)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(5/2),x)
```

```
[Out] 2/3*(-2*arcsinh(a*x)^(3/2)*Pi^(1/2)*x*a+arcsinh(a*x)^2*Pi*erf(arcsinh(a*x)^
(1/2))+arcsinh(a*x)^2*Pi*erfi(arcsinh(a*x)^(1/2))-arcsinh(a*x)^(1/2)*Pi^(1/
2)*(a^2*x^2+1)^(1/2))/Pi^(1/2)/a/arcsinh(a*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a*x)^(5/2),x)
```

```
[Out] int(1/asinh(a*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**(-5/2), x)
```

3.110 $\int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sinh^{-1}(ax)^{5/2}}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^(5/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^(5/2)),x]
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx = \int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx$$

Mathematica [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh^{-1}(ax)^{5/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^(5/2)),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^(5/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^(5/2)), x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(ax)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^(5/2),x)
[Out] int(1/x/arcsinh(a*x)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(a*x)^(5/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asinh}(a\,x)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)^(5/2)),x)
[Out] int(1/(x*asinh(a*x)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^{\frac{5}{2}}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**(5/2),x)
[Out] Integral(1/(x*asinh(a*x)**(5/2)), x)
```

3.111 $\int \frac{x^4}{\sinh^{-1}(ax)^{7/2}} dx$

Optimal. Leaf size=285

$$-\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{30a^5}+\frac{9\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{20a^5}-\frac{5\sqrt{5\pi}\,\text{erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}+\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{30a^5}$$

```
[Out] -16/15*x^3/a^2/arcsinh(a*x)^(3/2)-4/3*x^5/arcsinh(a*x)^(3/2)-1/30*erf(arcsi
nh(a*x)^(1/2))*Pi^(1/2)/a^5+1/30*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5+9/20
*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-9/20*erfi(3^(1/2)*arc
sinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-5/12*erf(5^(1/2)*arcsinh(a*x)^(1/2))*
5^(1/2)*Pi^(1/2)/a^5+5/12*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)
/a^5-2/5*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)-32/5*x^2*(a^2*x^2+1)^(1
/2)/a^3/arcsinh(a*x)^(1/2)-40/3*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5667, 5774, 5665, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{30a^5}+\frac{9\sqrt{3\pi}\,\text{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{20a^5}-\frac{5\sqrt{5\pi}\,\text{Erf}\left(\sqrt{5}\sqrt{\sinh^{-1}(ax)}\right)}{12a^5}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{30a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (16*x^3)/(15*a^2*ArcS
inh[a*x]^(3/2)) - (4*x^5)/(3*ArcSinh[a*x]^(3/2)) - (32*x^2*Sqrt[1 + a^2*x^2
])/(5*a^3*Sqrt[ArcSinh[a*x]]) - (40*x^4*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[ArcSin
h[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(30*a^5) + (9*Sqrt[3*Pi]*Erf[
Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(20*a^5) - (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcS
inh[a*x]]])/(12*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(30*a^5) - (9*Sq
rt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(20*a^5) + (5*Sqrt[5*Pi]*Erfi[Sq
rt[5]*Sqrt[ArcSinh[a*x]]])/(12*a^5)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^4}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{8\int \frac{x^3}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}\,dx}{5a} + (2a)\int \frac{x^5}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}\,dx$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} + \frac{20}{3}\int \frac{x^4}{\sinh^{-1}(ax)^{3/2}}\,dx + \cdots$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{\cdots}}{3a\sqrt{\sinh^{-1}(ax)}}\cdots$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{\cdots}}{3a\sqrt{\sinh^{-1}(ax)}}\cdots$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{\cdots}}{3a\sqrt{\sinh^{-1}(ax)}}\cdots$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{\cdots}}{3a\sqrt{\sinh^{-1}(ax)}}\cdots$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{16x^3}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^5}{3\sinh^{-1}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{40x^4\sqrt{\cdots}}{3a\sqrt{\sinh^{-1}(ax)}}\cdots$$

Mathematica [A] time = 0.75, size = 334, normalized size = 1.17

$$-2e^{\sinh^{-1}(ax)}\left(4\sinh^{-1}(ax)^2+2\sinh^{-1}(ax)+3\right)+9e^{3\sinh^{-1}(ax)}\left(12\sinh^{-1}(ax)^2+2\sinh^{-1}(ax)+1\right)-e^{5\sinh^{-1}(ax)}\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) + 9*E^(3*ArcSinh
[a*x])*(1 + 2*ArcSinh[a*x] + 12*ArcSinh[a*x]^2) - E^(5*ArcSinh[a*x])*(3 + 1
0*ArcSinh[a*x] + 100*ArcSinh[a*x]^2) + 100*Sqrt[5]*(-ArcSinh[a*x])^(5/2)*Ga
mma[1/2, -5*ArcSinh[a*x]] - 108*Sqrt[3]*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -3
*ArcSinh[a*x]] + 8*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -ArcSinh[a*x]] + (-6 +
4*ArcSinh[a*x] - 8*ArcSinh[a*x]^2 + 8*E^ArcSinh[a*x]*ArcSinh[a*x]^(5/2)*Gam
ma[1/2, ArcSinh[a*x]])/E^ArcSinh[a*x] + (9*(1 - 2*ArcSinh[a*x] + 12*ArcSinh
[a*x]^2 - 12*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*Gamma[1/2, 3*Arc
Sinh[a*x]]))/E^(3*ArcSinh[a*x]) + (-3 + 10*ArcSinh[a*x] - 100*ArcSinh[a*x]^
2 + 100*Sqrt[5]*E^(5*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*Gamma[1/2, 5*ArcSinh[
a*x]])/E^(5*ArcSinh[a*x]))/(240*a^5*ArcSinh[a*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="maxima")
[Out] integrate(x^4/arcsinh(a*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\operatorname{asinh}(a\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/asinh(a*x)^(7/2),x)
[Out] int(x^4/asinh(a*x)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)**(7/2),x)
[Out] Integral(x**4/asinh(a*x)**(7/2), x)
```

3.112 $\int \frac{x^3}{\sinh^{-1}(ax)^{7/2}}\, dx$

Optimal. Leaf size=229

$$\frac{16\sqrt{\pi}\,\mathrm{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}+\frac{16\sqrt{\pi}\,\mathrm{erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\sinh^{-}\right.}}{15a^4}$$

```
[Out] -4/5*x^2/a^2/arcsinh(a*x)^(3/2)-16/15*x^4/arcsinh(a*x)^(3/2)+16/15*erf(2*ar
csinh(a*x)^(1/2))*Pi^(1/2)/a^4+16/15*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^
4-4/15*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-4/15*erfi(2^(1/
2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-2/5*x^3*(a^2*x^2+1)^(1/2)/a/arc
sinh(a*x)^(5/2)-16/5*x*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)^(1/2)-128/15*x^3*
(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5667, 5774, 5665, 3307, 2180, 2204, 2205}

$$\frac{16\sqrt{\pi}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}+\frac{16\sqrt{\pi}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh\right.}}{15a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (4*x^2)/(5*a^2*ArcSin
h[a*x]^(3/2)) - (16*x^4)/(15*ArcSinh[a*x]^(3/2)) - (16*x*Sqrt[1 + a^2*x^2])
/(5*a^3*Sqrt[ArcSinh[a*x]]) - (128*x^3*Sqrt[1 + a^2*x^2])/(15*a*Sqrt[ArcSin
h[a*x]]) + (16*Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(15*a^4) - (4*Sqrt[2*Pi]
*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^4) + (16*Sqrt[Pi]*Erfi[2*Sqrt[ArcSi
nh[a*x]]])/(15*a^4) - (4*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a
^4)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^3}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{6\int \frac{x^2}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(8a)\int \frac{x^4}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}\,dx$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} + \frac{64}{15}\int \frac{x^3}{\sinh^{-1}(ax)^{3/2}}\,dx + \cdots$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\cdots}{15a\sqrt{\cdots}}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\cdots}{15a\sqrt{\cdots}}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\cdots}{15a\sqrt{\cdots}}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\cdots}{15a\sqrt{\cdots}}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x^2}{5a^2\sinh^{-1}(ax)^{3/2}} - \frac{16x^4}{15\sinh^{-1}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\sinh^{-1}(ax)}} - \frac{128x^3\cdots}{15a\sqrt{\cdots}}$$

Mathematica [A] time = 0.73, size = 210, normalized size = 0.92

$$\frac{6\sinh\left(2\sinh^{-1}(ax)\right)-3\sinh\left(4\sinh^{-1}(ax)\right)+4\sinh^{-1}(ax)\left(e^{-2\sinh^{-1}(ax)}\left(1-4\sinh^{-1}(ax)\right)+e^{2\sinh^{-1}(ax)}\left(4\sinh\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (4*ArcSinh[a*x]*((1 - 4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x
])*(1 + 4*ArcSinh[a*x]) + 4*Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*Arc
Sinh[a*x]] + 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]) - 4*A
rcSinh[a*x]*((1 - 8*ArcSinh[a*x])/E^(4*ArcSinh[a*x]) + E^(4*ArcSinh[a*x])*(
1 + 8*ArcSinh[a*x]) + 16*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]]
+ 16*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]]) + 6*Sinh[2*ArcSinh[a*x]
] - 3*Sinh[4*ArcSinh[a*x]])/(60*a^4*ArcSinh[a*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\operatorname{asinh}(a\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/asinh(a*x)^(7/2),x)
[Out] int(x^3/asinh(a*x)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)**(7/2),x)
[Out] Integral(x**3/asinh(a*x)**(7/2), x)
```

3.113 $\int \frac{x^2}{\sinh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=222

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a^3}-\frac{3\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{5a^3}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\,\mathrm{erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{5a^3}$$

```
[Out] -8/15*x/a^2/arcsinh(a*x)^(3/2)-4/5*x^3/arcsinh(a*x)^(3/2)+1/15*erf(arcsinh(
a*x)^(1/2))*Pi^(1/2)/a^3-1/15*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3-3/5*erf
(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+3/5*erfi(3^(1/2)*arcsinh(
a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-2/5*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(
5/2)-16/15*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)^(1/2)-24/5*x^2*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5667, 5774, 5665, 3308, 2180, 2204, 2205, 5655, 5779}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a^3}-\frac{3\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{5a^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{\sinh^{-1}(ax)}\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (8*x)/(15*a^2*ArcSinh
[a*x]^(3/2)) - (4*x^3)/(5*ArcSinh[a*x]^(3/2)) - (16*Sqrt[1 + a^2*x^2])/(15*
a^3*Sqrt[ArcSinh[a*x]]) - (24*x^2*Sqrt[1 + a^2*x^2])/(5*a*Sqrt[ArcSinh[a*x]
]) + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(15*a^3) - (3*Sqrt[3*Pi]*Erf[Sqrt[3
]*Sqrt[ArcSinh[a*x]]])/(5*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(15*a^
3) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(5*a^3)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{4\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(6a)\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} + \frac{12}{5}\int \frac{x^2}{\sinh^{-1}(ax)^{3/2}}\,dx + \frac{8\int}{}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1}}{5a\sqrt{\sinh}}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1}}{5a\sqrt{\sinh}}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1}}{5a\sqrt{\sinh}}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1}}{5a\sqrt{\sinh}}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{8x}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{4x^3}{5\sinh^{-1}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\sinh^{-1}(ax)}} - \frac{24x^2\sqrt{1}}{5a\sqrt{\sinh}}$$

Mathematica [A] time = 0.40, size = 221, normalized size = 1.00

$$\frac{e^{\sinh^{-1}(ax)}\left(4\sinh^{-1}(ax)^2+2\sinh^{-1}(ax)+3\right)-3e^{3\sinh^{-1}(ax)}\left(12\sinh^{-1}(ax)^2+2\sinh^{-1}(ax)+1\right)+36\sqrt{3}\left(-\sinh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) - 3*E^(3*ArcSinh[a*
x])*(1 + 2*ArcSinh[a*x] + 12*ArcSinh[a*x]^2) + 36*Sqrt[3]*(-ArcSinh[a*x])^(
5/2)*Gamma[1/2, -3*ArcSinh[a*x]] - 4*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -ArcS
inh[a*x]] + (3 - 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2 - 4*E^ArcSinh[a*x]*ArcSi
nh[a*x]^(5/2)*Gamma[1/2, ArcSinh[a*x]])/E^ArcSinh[a*x] + (-3 + 6*ArcSinh[a*
x] - 36*ArcSinh[a*x]^2 + 36*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*G
amma[1/2, 3*ArcSinh[a*x]])/E^(3*ArcSinh[a*x]))/(60*a^3*ArcSinh[a*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(x^2/arcsinh(a*x)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)^(7/2),x)
[Out] int(x^2/arcsinh(a*x)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="maxima")
[Out] integrate(x^2/arcsinh(a*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\operatorname{asinh}(a\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a*x)^(7/2),x)
[Out] int(x^2/asinh(a*x)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)**(7/2),x)
[Out] Integral(x**2/asinh(a*x)**(7/2), x)
```

3.114 $\int \frac{x}{\sinh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=147

$$\frac{8\sqrt{2\pi}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^2}+\frac{8\sqrt{2\pi}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^2}-\frac{32x\sqrt{a^2x^2+1}}{15a\sqrt{\sinh^{-1}(ax)}}-\frac{2x\sqrt{a^2x^2+1}}{5a\sinh^{-1}(ax)^{5/2}}-\frac{4}{15a^2\sinh^{-1}(}$$

```
[Out] -4/15/a^2/arcsinh(a*x)^(3/2)-8/15*x^2/arcsinh(a*x)^(3/2)+8/15*erf(2^(1/2)*a
rcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+8/15*erfi(2^(1/2)*arcsinh(a*x)^(1/2
))*2^(1/2)*Pi^(1/2)/a^2-2/5*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)-32/15*
x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5667, 5774, 5665, 3307, 2180, 2204, 2205, 5675}

$$\frac{8\sqrt{2\pi}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^2}+\frac{8\sqrt{2\pi}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{15a^2}-\frac{32x\sqrt{a^2x^2+1}}{15a\sqrt{\sinh^{-1}(ax)}}-\frac{2x\sqrt{a^2x^2+1}}{5a\sinh^{-1}(ax)^{5/2}}-\frac{4}{15a^2\sinh^{-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - 4/(15*a^2*ArcSinh[a*x]^
(3/2)) - (8*x^2)/(15*ArcSinh[a*x]^(3/2)) - (32*x*Sqrt[1 + a^2*x^2])/(15*a*S
qrt[ArcSinh[a*x]]) + (8*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^2
) + (8*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^2)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
```

```
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{2\int \frac{1}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(4a)\int \frac{x^2}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}\,dx$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} + \frac{16}{15}\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{32\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{16\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{32\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4}{15a^2\sinh^{-1}(ax)^{3/2}} - \frac{8x^2}{15\sinh^{-1}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{8\sqrt{2\cdots}}{\cdots}$$

Mathematica [A] time = 0.35, size = 118, normalized size = 0.80

$$-\frac{3\sinh\left(2\sinh^{-1}(ax)\right) + 2\sinh^{-1}(ax)\left(e^{-2\sinh^{-1}(ax)}\left(1-4\sinh^{-1}(ax)\right) + e^{2\sinh^{-1}(ax)}\left(4\sinh^{-1}(ax)+1\right) + 4\sqrt{2}\cdots\right.}{15a^2\sinh^{-1}(ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/ArcSinh[a*x]^(7/2),x]
```

```
[Out] -1/15*(2*ArcSinh[a*x]*((1 - 4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]) + E^(2*ArcSi
nh[a*x])*(1 + 4*ArcSinh[a*x]) + 4*Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2,
-2*ArcSinh[a*x]] + 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]])
 + 3*Sinh[2*ArcSinh[a*x]])/(a^2*ArcSinh[a*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^(7/2), x)
```

maple [A] time = 0.34, size = 147, normalized size = 1.00

$$\frac{\sqrt{2}\left(-16 \operatorname{arcsinh}(ax)^{\frac{5}{2}} \sqrt{2}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,xa - 4 \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{2}\,\sqrt{\pi}\,x^2a^2 - 3\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^2x^2+\right.}{15\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)^(7/2),x)
```

```
[Out] 1/15*2^(1/2)*(-16*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a
-4*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*x^2*a^2-3*2^(1/2)*arcsinh(a*x)^(1/2)
*Pi^(1/2)*(a^2*x^2+1)^(1/2)*x*a+8*arcsinh(a*x)^3*Pi*erf(2^(1/2)*arcsinh(a*x
)^(1/2))+8*arcsinh(a*x)^3*Pi*erfi(2^(1/2)*arcsinh(a*x)^(1/2))-2*arcsinh(a*x
)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arcsinh(a*x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asinh}(a\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a*x)^(7/2),x)
```

```
[Out] int(x/asinh(a*x)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)**(7/2),x)
```

```
[Out] Integral(x/asinh(a*x)**(7/2), x)
```

3.115 $\int \frac{1}{\sinh^{-1}(ax)^{7/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{8\sqrt{a^2x^2+1}}{15a\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x^2+1}}{5a\sinh^{-1}(ax)^{5/2}}-\frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a}+\frac{4\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a}-\frac{4x}{15\sinh^{-1}(ax)^{3/2}}$$

```
[Out] -4/15*x/arcsinh(a*x)^(3/2)-4/15*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+4/15*erf
i(arcsinh(a*x)^(1/2))*Pi^(1/2)/a-2/5*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)
-8/15*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5655, 5774, 5779, 3308, 2180, 2204, 2205}

$$-\frac{8\sqrt{a^2x^2+1}}{15a\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x^2+1}}{5a\sinh^{-1}(ax)^{5/2}}-\frac{4\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a}+\frac{4\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a}-\frac{4x}{15\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (4*x)/(15*ArcSinh[a*x]^(3
/2)) - (8*Sqrt[1 + a^2*x^2])/(15*a*Sqrt[ArcSinh[a*x]]) - (4*Sqrt[Pi]*Erf[Sq
rt[ArcSinh[a*x]]])/(15*a) + (4*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(15*a)
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(ax)^{7/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} + \frac{1}{5}(2a)\int \frac{x}{\sqrt{1+a^2x^2}\,\sinh^{-1}(ax)^{5/2}}\,dx$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} + \frac{4}{15}\int \frac{1}{\sinh^{-1}(ax)^{3/2}}\,dx$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{1}{15}(8a)\int \frac{x}{\sqrt{1+a^2x^2}\,\sqrt{\sin}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} + \frac{8\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sin\right.}{15a}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} - \frac{4\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^-\right.}{15a}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} - \frac{8\,\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sin}\right.}{15a}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{5a\sinh^{-1}(ax)^{5/2}} - \frac{4x}{15\sinh^{-1}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\sinh^{-1}(ax)}} - \frac{4\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\sinh^{-1}(ax)}\right)}{15a} +$$

Mathematica [A] time = 0.20, size = 111, normalized size = 0.99

$$\frac{-2e^{\sinh^{-1}(ax)}\left(4\sinh^{-1}(ax)^2+2\sinh^{-1}(ax)+3\right)+8\left(-\sinh^{-1}(ax)\right)^{5/2}\Gamma\left(\frac{1}{2},-\sinh^{-1}(ax)\right)+e^{-\sinh^{-1}(ax)}\left(-8\sin\right.}{30a\sinh^{-1}(ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^(-7/2),x]
```

```
[Out] (-2*E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) + 8*(-ArcSinh[a*
x])^(5/2)*Gamma[1/2, -ArcSinh[a*x]] + (-6 + 4*ArcSinh[a*x] - 8*ArcSinh[a*x]
```

```
^2 + 8*E^ArcSinh[a*x]*ArcSinh[a*x]^(5/2)*Gamma[1/2, ArcSinh[a*x]])/E^ArcSin
h[a*x])/(30*a*ArcSinh[a*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-7/2), x)
```

maple [A] time = 0.33, size = 105, normalized size = 0.94

$$-\frac{2\left(2 \operatorname{arcsinh}(ax)^{3} \pi \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-2 \operatorname{arcsinh}(ax)^{3} \pi \operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)+4\sqrt{a^{2}x^{2}+1}\,\sqrt{\pi}\,\operatorname{arcsinh}(\right.}{15\sqrt{\pi}\, a \operatorname{arcsinh}(ax)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(7/2),x)
```

```
[Out] -2/15*(2*arcsinh(a*x)^3*Pi*erf(arcsinh(a*x)^(1/2))-2*arcsinh(a*x)^3*Pi*erfi
(arcsinh(a*x)^(1/2))+4*(a^2*x^2+1)^(1/2)*Pi^(1/2)*arcsinh(a*x)^(5/2)+2*arcs
inh(a*x)^(3/2)*Pi^(1/2)*x*a+3*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)
)/Pi^(1/2)/a/arcsinh(a*x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{asinh}(a\,x)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a*x)^(7/2),x)
```

```
[Out] int(1/asinh(a*x)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(7/2),x)
```

```
[Out] Integral(asinh(a*x)**(-7/2), x)
```

3.116 $\int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sinh^{-1}(ax)^{7/2}}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^(7/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a*x]^(7/2)),x]
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(7/2)), x]
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx = \int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx$$

Mathematica [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh^{-1}(ax)^{7/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a*x]^(7/2)),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^(7/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^(7/2)), x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(ax)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)^(7/2),x)
[Out] int(1/x/arcsinh(a*x)^(7/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arsinh}(ax)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(a*x)^(7/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asinh}(a\,x)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a*x)^(7/2)),x)
[Out] int(1/(x*asinh(a*x)^(7/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^{\frac{7}{2}}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)**(7/2),x)
[Out] Integral(1/(x*asinh(a*x)**(7/2)), x)
```

3.117 $\int x^m \sinh^{-1}(ax)^4\,dx$

Optimal. Leaf size=54

$$\frac{x^{m+1}\sinh^{-1}(ax)^4}{m+1}-\frac{4a\mathrm{Int}\left(\frac{x^{m+1}\sinh^{-1}(ax)^3}{\sqrt{a^2x^2+1}},x\right)}{m+1}$$

```
[Out] x^(1+m)*arcsinh(a*x)^4/(1+m)-4*a*Unintegrable(x^(1+m)*arcsinh(a*x)^3/(a^2*x
^2+1)^(1/2),x)/(1+m)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sinh^{-1}(ax)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcSinh[a*x]^4,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^4)/(1 + m) - (4*a*Defer[Int][(x^(1 + m)*ArcSinh[a*x
]^3)/Sqrt[1 + a^2*x^2], x])/(1 + m)
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^4\,dx=\frac{x^{1+m}\sinh^{-1}(ax)^4}{1+m}-\frac{(4a)\int\frac{x^{1+m}\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{1+m}$$

Mathematica [A] time = 0.88, size = 0, normalized size = 0.00

$$\int x^m \sinh^{-1}(ax)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcSinh[a*x]^4,x]
```

```
[Out] Integrate[x^m*ArcSinh[a*x]^4, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m\,\mathrm{arsinh}\,(ax)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\,\mathrm{arsinh}\,(ax)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^4, x)
```

maple [A] time = 1.32, size = 0, normalized size = 0.00

$$\int x^{m} \operatorname{arcsinh}(ax)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^4,x)
```

```
[Out] int(x^m*arcsinh(a*x)^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xx^{m}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{4}}{m+1}-\int\frac{4\left(\sqrt{a^{2}x^{2}+1}\,a^{2}x^{2}x^{m}+\left(a^{3}x^{3}+ax\right)x^{m}\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}}{a^{3}(m+1)x^{3}+a(m+1)x+\left(a^{2}(m+1)x^{2}+m+1\right)\sqrt{a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^4/(m + 1) - integrate(4*(sqrt(a^2*x^2 +
1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*(
m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m}\,\operatorname{asinh}(a\,x)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x)^4,x)
```

```
[Out] int(x^m*asinh(a*x)^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\operatorname{asinh}^{4}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**4,x)
```

```
[Out] Integral(x**m*asinh(a*x)**4, x)
```

3.118 $\int x^m \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=54

$$\frac{x^{m+1}\sinh^{-1}(ax)^3}{m+1} - \frac{3a\mathrm{Int}\left(\frac{x^{m+1}\sinh^{-1}(ax)^2}{\sqrt{a^2x^2+1}}, x\right)}{m+1}$$

```
[Out] x^(1+m)*arcsinh(a*x)^3/(1+m)-3*a*Unintegrable(x^(1+m)*arcsinh(a*x)^2/(a^2*x
^2+1)^(1/2),x)/(1+m)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sinh^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcSinh[a*x]^3,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^3)/(1 + m) - (3*a*Defer[Int][(x^(1 + m)*ArcSinh[a*x
]^2)/Sqrt[1 + a^2*x^2], x])/(1 + m)
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^3 \, dx = \frac{x^{1+m}\sinh^{-1}(ax)^3}{1+m} - \frac{(3a)\int \frac{x^{1+m}\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx}{1+m}$$

Mathematica [A] time = 0.79, size = 0, normalized size = 0.00

$$\int x^m \sinh^{-1}(ax)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcSinh[a*x]^3,x]
```

```
[Out] Integrate[x^m*ArcSinh[a*x]^3, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m \operatorname{arsinh}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arsinh}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^3, x)
```

maple [A] time = 1.19, size = 0, normalized size = 0.00

$$\int x^{m} \operatorname{arcsinh}(ax)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^3,x)
```

```
[Out] int(x^m*arcsinh(a*x)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xx^{m}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}}{m+1}-\int \frac{3\left(\sqrt{a^{2}x^{2}+1}\,a^{2}x^{2}x^{m}+\left(a^{3}x^{3}+ax\right)x^{m}\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}}{a^{3}(m+1)x^{3}+a(m+1)x+\left(a^{2}(m+1)x^{2}+m+1\right)\sqrt{a^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^3/(m + 1) - integrate(3*(sqrt(a^2*x^2 +
1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*(
m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m}\,\operatorname{asinh}(a\,x)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x)^3,x)
```

```
[Out] int(x^m*asinh(a*x)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \operatorname{asinh}^{3}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**3,x)
```

```
[Out] Integral(x**m*asinh(a*x)**3, x)
```

3.119 $\int x^m \sinh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=137

$$\frac{2a^2x^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-a^2x^2\right)}{m^3+6m^2+11m+6}-\frac{2ax^{m+2}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}+\frac{x^{m+1}\sinh^{-1}(a}{m+1}$$

```
[Out] x^(1+m)*arcsinh(a*x)^2/(1+m)-2*a*x^(2+m)*arcsinh(a*x)*hypergeom([1/2, 1+1/2
*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)+2*a^2*x^(3+m)*HypergeometricPFQ([1, 3/2
+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],-a^2*x^2)/(m^3+6*m^2+11*m+6)
```

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5661, 5762}

$$\frac{2a^2x^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-a^2x^2\right)}{m^3+6m^2+11m+6}-\frac{2ax^{m+2}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}+\frac{x^{m+1}\sinh^{-1}(a}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcSinh[a*x]^2,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^2)/(1 + m) - (2*a*x^(2 + m)*ArcSinh[a*x]*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) + (2*a^2*x^
(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
-(a^2*x^2)])/(6 + 11*m + 6*m^2 + m^3)
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^m \sinh^{-1}(ax)^2\,dx &= \frac{x^{1+m}\sinh^{-1}(ax)^2}{1+m}-\frac{(2a)\int \frac{x^{1+m}\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{1+m}\\ &= \frac{x^{1+m}\sinh^{-1}(ax)^2}{1+m}-\frac{2ax^{2+m}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}+\frac{2a^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2}\right.}{6+11}\end{aligned}$$

Mathematica [A] time = 0.04, size = 123, normalized size = 0.90

$$\frac{x^{m+1}\left(2a^2x^2\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-a^2x^2\right)+(m+3)\sinh^{-1}(ax)\left((m+2)\sinh^{-1}(ax)-2ax\,{}_2F_1\left(\frac{1}{2},\frac{m}{}\right.\right.\right.}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcSinh[a*x]^2,x]
```

```
[Out] (x^(1 + m)*((3 + m)*ArcSinh[a*x]*((2 + m)*ArcSinh[a*x] - 2*a*x*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]) + 2*a^2*x^2*HypergeometricPFQ
[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, -(a^2*x^2)]))/((1 + m)*(2
 + m)*(3 + m))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \operatorname{arsinh}\left(ax\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arsinh}\left(ax\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^2, x)
```

maple [F] time = 1.21, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arcsinh}\left(ax\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^2,x)
```

```
[Out] int(x^m*arcsinh(a*x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xx^m \log\left(ax + \sqrt{a^2x^2 + 1}\right)^2}{m + 1} - \int \frac{2\left(\sqrt{a^2x^2 + 1}\, a^2x^2x^m + \left(a^3x^3 + ax\right)x^m\right)\log\left(ax + \sqrt{a^2x^2 + 1}\right)}{a^3(m + 1)x^3 + a(m + 1)x + \left(a^2(m + 1)x^2 + m + 1\right)\sqrt{a^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^2/(m + 1) - integrate(2*(sqrt(a^2*x^2 +
1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*(m
+ 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \operatorname{asinh}\left(a\,x\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x)^2,x)
```

```
[Out] int(x^m*asinh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{asinh}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**2,x)
```

```
[Out] Integral(x**m*asinh(a*x)**2, x)
```

3.120 $\int x^m \sinh^{-1}(ax)\, dx$

Optimal. Leaf size=60

$$\frac{x^{m+1}\sinh^{-1}(ax)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}$$

```
[Out] x^(1+m)*arcsinh(a*x)/(1+m)-a*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^
2*x^2)/(m^2+3*m+2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5661, 364}

$$\frac{x^{m+1}\sinh^{-1}(ax)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcSinh[a*x],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x])/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 +
 m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m\sinh^{-1}(ax)\,dx&=\frac{x^{1+m}\sinh^{-1}(ax)}{1+m}-\frac{a\int\frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx}{1+m}\\&=\frac{x^{1+m}\sinh^{-1}(ax)}{1+m}-\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.92

$$\frac{x^{m+1}\left((m+2)\sinh^{-1}(ax)-ax\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcSinh[a*x],x]
```

```
[Out] (x^(1 + m)*((2 + m)*ArcSinh[a*x] - a*x*Hypergeometric2F1[1/2, (2 + m)/2, (4
 + m)/2, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \operatorname{arsinh}\left(ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arsinh}\left(ax\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x), x)
```

maple [F] time = 1.20, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arcsinh}\left(ax\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x),x)
```

```
[Out] int(x^m*arcsinh(a*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2 \int \frac{x^2 x^m}{a^2(m+1)x^2 + m + 1} \, dx - a \int \frac{x x^m}{a^3(m+1)x^3 + a(m+1)x + \left(a^2(m+1)x^2 + m + 1\right)\sqrt{a^2x^2 + 1}} \, dx + \frac{x x^m \log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] -a^2*integrate(x^2*x^m/(a^2*(m + 1)*x^2 + m + 1), x) - a*integrate(x*x^m/(a
^3*(m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1))
, x) + x*x^m*log(a*x + sqrt(a^2*x^2 + 1))/(m + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \operatorname{asinh}\left(a\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x),x)
```

```
[Out] int(x^m*asinh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \operatorname{asinh}{\left(ax \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x),x)
```

```
[Out] Integral(x**m*asinh(a*x), x)
```

3.121 $\int \frac{x^m}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x^m}{\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(x^m/arcsinh(a*x),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcSinh[a*x],x]
[Out] Defer[Int][x^m/ArcSinh[a*x], x]
```

Rubi steps

$$\int \frac{x^m}{\sinh^{-1}(ax)}\,dx = \int \frac{x^m}{\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcSinh[a*x],x]
[Out] Integrate[x^m/ArcSinh[a*x], x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x^m/arcsinh(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(x^m/arcsinh(a*x), x)
```

maple [A] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arcsinh(a*x),x)
[Out] int(x^m/arcsinh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(x^m/arcsinh(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^m}{\operatorname{asinh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/asinh(a*x),x)
[Out] int(x^m/asinh(a*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/asinh(a*x),x)
[Out] Integral(x**m/asinh(a*x), x)
```

3.122 $\int \frac{x^m}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x^m}{\sinh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable(x^m/arcsinh(a*x)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcSinh[a*x]^2,x]
[Out] Defer[Int][x^m/ArcSinh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x^m}{\sinh^{-1}(ax)^2}\,dx = \int \frac{x^m}{\sinh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcSinh[a*x]^2,x]
[Out] Integrate[x^m/ArcSinh[a*x]^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\operatorname{arsinh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(x^m/arcsinh(a*x)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate(x^m/arcsinh(a*x)^2, x)
```

maple [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arcsinh(a*x)^2,x)
```

```
[Out] int(x^m/arcsinh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x^m+\left(a^3x^3+ax\right)x^m}{\left(a^3x^2+\sqrt{a^2x^2+1}\,a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}+\int\frac{\left(a^3(m+1)x^3+a(m-1)x\right)\left(a^2x^2+1\right)x^m+\left(2\,a^4(m+1)x^4\right.}{\left(a^5x^5+\left(a^2x^2+1\right)a^3x^3+2\,a^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^2*x^2 + 1)^(3/2)*x^m + (a^3*x^3 + a*x)*x^m)/((a^3*x^2 + sqrt(a^2*x^2 +
 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate(((a^3*(m + 1)*x^3
+ a*(m - 1)*x)*(a^2*x^2 + 1)*x^m + (2*a^4*(m + 1)*x^4 + a^2*(3*m + 1)*x^2 +
 m)*sqrt(a^2*x^2 + 1)*x^m + (a^5*(m + 1)*x^5 + 2*a^3*(m + 1)*x^3 + a*(m + 1
)*x)*x^m)/((a^5*x^5 + (a^2*x^2 + 1)*a^3*x^3 + 2*a^3*x^3 + a*x + 2*(a^4*x^4
+ a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^m}{\operatorname{asinh}(a\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/asinh(a*x)^2,x)
```

```
[Out] int(x^m/asinh(a*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{asinh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/asinh(a*x)**2,x)
```

```
[Out] Integral(x**m/asinh(a*x)**2, x)
```

3.123 $\int x^m \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(x^m \sinh^{-1}(ax)^{5/2}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^(5/2),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sinh^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcSinh[a*x]^(5/2),x]
[Out] Defer[Int][x^m*ArcSinh[a*x]^(5/2), x]
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^{5/2}\,dx = \int x^m \sinh^{-1}(ax)^{5/2}\,dx$$

Mathematica [A] time = 1.12, size = 0, normalized size = 0.00

$$\int x^m \sinh^{-1}(ax)^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcSinh[a*x]^(5/2),x]
[Out] Integrate[x^m*ArcSinh[a*x]^(5/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arcsinh}(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int x^m \operatorname{asinh}(a\,x)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x)^(5/2),x)
```

```
[Out] int(x^m*asinh(a*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

3.124 $\int x^m \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(x^m \sinh^{-1}(ax)^{3/2}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^(3/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sinh^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*ArcSinh[a*x]^(3/2),x]
[Out] Defer[Int][x^m*ArcSinh[a*x]^(3/2), x]
```

Rubi steps

$$\int x^m \sinh^{-1}(ax)^{3/2}\,dx = \int x^m \sinh^{-1}(ax)^{3/2}\,dx$$

Mathematica [A] time = 1.12, size = 0, normalized size = 0.00

$$\int x^m \sinh^{-1}(ax)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*ArcSinh[a*x]^(3/2),x]
[Out] Integrate[x^m*ArcSinh[a*x]^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arcsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int x^m \operatorname{asinh}(a\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x)^(3/2),x)
```

```
[Out] int(x^m*asinh(a*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)**(3/2), x)
```

3.125 $\int x^m \sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(x^m\sqrt{\sinh^{-1}(ax)},x\right)$$

[Out] Unintegrable(x^m*arcsinh(a*x)^(1/2),x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\sqrt{\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Int[x^m*Sqrt[ArcSinh[a*x]],x]

[Out] Defer[Int][x^m*Sqrt[ArcSinh[a*x]], x]

Rubi steps

$$\int x^m\sqrt{\sinh^{-1}(ax)}\,dx=\int x^m\sqrt{\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.41, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m*Sqrt[ArcSinh[a*x]],x]

[Out] Integrate[x^m*Sqrt[ArcSinh[a*x]], x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\operatorname{arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\operatorname{arsinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt(arcsinh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int x^m\, \sqrt{\operatorname{asinh}(a\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x)^(1/2),x)
```

```
[Out] int(x^m*asinh(a*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\operatorname{asinh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(asinh(a*x)), x)
```

3.126 $\int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x^m}{\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m/arcsinh(a*x)^(1/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Defer[Int][x^m/Sqrt[ArcSinh[a*x]], x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Integrate[x^m/Sqrt[ArcSinh[a*x]], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(arcsinh(a*x)), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arcsinh(a*x)^(1/2),x)
[Out] int(x^m/arcsinh(a*x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt(arcsinh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^m}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/asinh(a*x)^(1/2),x)
[Out] int(x^m/asinh(a*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/asinh(a*x)**(1/2),x)
[Out] Integral(x**m/sqrt(asinh(a*x)), x)
```

3.127 $\int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x^m}{\sinh^{-1}(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x^m/arcsinh(a*x)^(3/2),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/ArcSinh[a*x]^(3/2),x]
[Out] Defer[Int][x^m/ArcSinh[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\, dx = \int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\, dx$$

Mathematica [A] time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sinh^{-1}(ax)^{3/2}}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/ArcSinh[a*x]^(3/2),x]
[Out] Integrate[x^m/ArcSinh[a*x]^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^m/arcsinh(a*x)^(3/2), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arcsinh(a*x)^(3/2),x)
[Out] int(x^m/arcsinh(a*x)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m/arcsinh(a*x)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^m}{\operatorname{asinh}(a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/asinh(a*x)^(3/2),x)
[Out] int(x^m/asinh(a*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/asinh(a*x)**(3/2),x)
[Out] Integral(x**m/asinh(a*x)**(3/2), x)
```

3.128 $\int (bx)^m \sinh^{-1}(ax)^n \, dx$

Optimal. Leaf size=15

$$\text{Int}\left((bx)^m \sinh^{-1}(ax)^n, x\right)$$

[Out] Unintegrable((b*x)^m*arcsinh(a*x)^n,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (bx)^m \sinh^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

[In] Int[(b*x)^m*ArcSinh[a*x]^n,x]

[Out] Defer[Int][(b*x)^m*ArcSinh[a*x]^n, x]

Rubi steps

$$\int (bx)^m \sinh^{-1}(ax)^n \, dx = \int (bx)^m \sinh^{-1}(ax)^n \, dx$$

Mathematica [A] time = 0.79, size = 0, normalized size = 0.00

$$\int (bx)^m \sinh^{-1}(ax)^n \, dx$$

Verification is Not applicable to the result.

[In] Integrate[(b*x)^m*ArcSinh[a*x]^n,x]

[Out] Integrate[(b*x)^m*ArcSinh[a*x]^n, x]

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((bx)^m \operatorname{arsinh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="fricas")

[Out] integral((b*x)^m*arcsinh(a*x)^n, x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int (bx)^m \operatorname{arcsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*arcsinh(a*x)^n,x)
```

```
[Out] int((b*x)^m*arcsinh(a*x)^n,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x)^m*arcsinh(a*x)^n, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.07

$$\int \operatorname{asinh}(a\,x)^n\,(b\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n*(b*x)^m,x)
```

```
[Out] int(asinh(a*x)^n*(b*x)^m, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx)^m \operatorname{asinh}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*asinh(a*x)**n,x)
```

```
[Out] Integral((b*x)**m*asinh(a*x)**n, x)
```

3.129 $\int x^4 \sinh^{-1}(ax)^n \, dx$

Optimal. Leaf size=173

$$\frac{5^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1,-5\sinh^{-1}(ax)\right)}{32a^5}-\frac{3^{-n}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1,-3\sinh\right.}{32a^5}$$

```
[Out] 1/32*5^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-5*arcsinh(a*x))/a^5/((-arcsinh(a*x)
)^n)-1/32*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/(3^n)/a^5/((-arcsinh(a*
x))^n)+1/16*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^5/((-arcsinh(a*x))^n)
-1/16*GAMMA(1+n,arcsinh(a*x))/a^5+1/32*GAMMA(1+n,3*arcsinh(a*x))/(3^n)/a^5-
1/32*5^(-1-n)*GAMMA(1+n,5*arcsinh(a*x))/a^5
```

Rubi [A] time = 0.22, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5669, 5448, 3307, 2181}

$$\frac{5^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-5\sinh^{-1}(ax)\right)}{32a^5}-\frac{3^{-n}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamm}}{32a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSinh[a*x]^n,x]
```

```
[Out] (5^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -5*ArcSinh[a*x]])/(32*a^5*(-ArcSinh
[a*x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(32*3^n*a^5*(-Ar
cSinh[a*x])^n) + (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(16*a^5*(-Arc
Sinh[a*x])^n) - Gamma[1 + n, ArcSinh[a*x]]/(16*a^5) + Gamma[1 + n, 3*ArcSin
h[a*x]]/(32*3^n*a^5) - (5^(-1 - n)*Gamma[1 + n, 5*ArcSinh[a*x]])/(32*a^5)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^4 \sinh^{-1}(ax)^n \, dx = \frac{\text{Subst}\left(\int x^n \cosh(x)\sinh^4(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{8}x^n\cosh(x) - \frac{3}{16}x^n\cosh(3x) + \frac{1}{16}x^n\cosh(5x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int x^n\cosh(5x)\,dx, x, \sinh^{-1}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int x^n\cosh(x)\,dx, x, \sinh^{-1}(ax)\right)}{8a^5} - \frac{3\,\text{Su}}{}$$
$$= \frac{\text{Subst}\left(\int e^{-5x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int e^{5x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int e^{-x}x^n\right.}{1}$$
$$= \frac{5^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n, -5\sinh^{-1}(ax)\right)}{32a^5} - \frac{3^{-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh}{}$$

Mathematica [A] time = 0.16, size = 145, normalized size = 0.84

$$\frac{5^{-n}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1, -5\sinh^{-1}(ax)\right) - 5\,3^{-n}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1, -3\sinh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSinh[a*x]^n,x]
```

```
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -5*ArcSinh[a*x]])/(5^n*(-ArcSinh[a*x])^n) - (
5*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(3^n*(-ArcSinh[a*x])^n) + (
10*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n - 10*Gamma
[1 + n, ArcSinh[a*x]] + (5*Gamma[1 + n, 3*ArcSinh[a*x]])/3^n - Gamma[1 + n,
 5*ArcSinh[a*x]]/5^n)/(160*a^5)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4 \operatorname{arsinh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^4*arcsinh(a*x)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^n, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arcsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^n,x)
```

```
[Out] int(x^4*arcsinh(a*x)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arsinh}(ax)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="maxima")
[Out] integrate(x^4*arcsinh(a*x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{asinh}(a\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*asinh(a*x)^n,x)
[Out] int(x^4*asinh(a*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{asinh}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**n,x)
[Out] Integral(x**4*asinh(a*x)**n, x)
```

3.130 $\int x^3 \sinh^{-1}(ax)^n \, dx$

Optimal. Leaf size=119

$$\frac{2^{-2(n+3)} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \Gamma\left(n+1,-4\sinh^{-1}(ax)\right)}{a^4} - \frac{2^{-n-4} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \Gamma\left(n+1,-2\right.}{a^4}$$

```
[Out] arcsinh(a*x)^n*GAMMA(1+n,-4*arcsinh(a*x))/(2^(6+2*n))/a^4/((-arcsinh(a*x))^
n)-2^(-4-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcsinh(a*x))/a^4/((-arcsinh(a*x))^
n)-2^(-4-n)*GAMMA(1+n,2*arcsinh(a*x))/a^4+GAMMA(1+n,4*arcsinh(a*x))/(2^(6+2
*n))/a^4
```

Rubi [A] time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5669, 5448, 3308, 2181}

$$\frac{2^{-2(n+3)} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-4\sinh^{-1}(ax)\right)}{a^4} - \frac{2^{-n-4} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gam}}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x]^n,x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]])/(2^(2*(3 + n))*a^4*(-ArcSinh
[a*x])^n) - (2^(-4 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^4*
(-ArcSinh[a*x])^n) - (2^(-4 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^4 + Gamma[
1 + n, 4*ArcSinh[a*x]]/(2^(2*(3 + n))*a^4)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3 \sinh^{-1}(ax)^n\,dx = \frac{\text{Subst}\left(\int x^n \cosh(x)\sinh^3(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}x^n \sinh(2x) + \frac{1}{8}x^n \sinh(4x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int x^n \sinh(4x)\,dx, x, \sinh^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int x^n \sinh(2x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{\text{Subst}\left(\int e^{-4x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int e^{4x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int e\right.}{}$$
$$= \frac{4^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n, -4\sinh^{-1}(ax)\right)}{a^4} - \frac{2^{-4-n}\left(-\sinh^{-1}(ax)\right)^{-n}}{}$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.83

$$\frac{4^{-n-3}\left(-\sinh^{-1}(ax)\right)^{-n}\left(\left(-\sinh^{-1}(ax)\right)^n\left(\Gamma\left(n+1, 4\sinh^{-1}(ax)\right) - 2^{n+2}\Gamma\left(n+1, 2\sinh^{-1}(ax)\right)\right) + \sinh^{-1}(ax)^n\right.}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x]^n,x]
```

```
[Out] (4^(-3 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]] - 2^(2 + n)*ArcSi
nh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(2^(2 + n)*Ga
mma[1 + n, 2*ArcSinh[a*x]]) + Gamma[1 + n, 4*ArcSinh[a*x]])))/(a^4*(-ArcSin
h[a*x])^n)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3 \operatorname{arsinh}(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^3*arcsinh(a*x)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arcsinh}(ax)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^n,x)
```

```
[Out] int(x^3*arcsinh(a*x)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{asinh}(a\,x)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x)^n,x)
```

```
[Out] int(x^3*asinh(a*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asinh}^n(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**n,x)
```

```
[Out] Integral(x**3*asinh(a*x)**n, x)
```

3.131 $\int x^2 \sinh^{-1}(ax)^n \, dx$

Optimal. Leaf size=113

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^3}-\frac{\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1,-\sinh^{-1}\right.}{8a^3}$$

```
[Out] 1/8*3^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/a^3/((-arcsinh(a*x))
^n)-1/8*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^3/((-arcsinh(a*x))^n)+1/8
*GAMMA(1+n,arcsinh(a*x))/a^3-1/8*3^(-1-n)*GAMMA(1+n,3*arcsinh(a*x))/a^3
```

Rubi [A] time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5669, 5448, 3307, 2181}

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^3}-\frac{\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x]^n,x]
```

```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^3*(-ArcSinh[
a*x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^3*(-ArcSinh[a*
x])^n) + Gamma[1 + n, ArcSinh[a*x]]/(8*a^3) - (3^(-1 - n)*Gamma[1 + n, 3*Ar
cSinh[a*x]])/(8*a^3)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2 \sinh^{-1}(ax)^n\,dx = \frac{\text{Subst}\left(\int x^n\cosh(x)\sinh^2(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{4}x^n\cosh(x)+\frac{1}{4}x^n\cosh(3x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int x^n\cosh(x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}+\frac{\text{Subst}\left(\int x^n\cosh(3x)\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}$$
$$= \frac{\text{Subst}\left(\int e^{-3x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{8a^3}-\frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{8a^3}-\frac{\text{Subst}\left(\int e^{x}x^n\,d\right.}{8}$$
$$= \frac{3^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-3\sinh^{-1}(ax)\right)}{8a^3}-\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(a}{8a}$$

Mathematica [A] time = 0.07, size = 97, normalized size = 0.86

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1,-3\sinh^{-1}(ax)\right)-\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1,-\sinh^{-1}(a\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x]^n,x]
```

```
[Out] ((3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(-ArcSinh[a*x])^
n - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamma[
1 + n, ArcSinh[a*x]] - 3^(-1 - n)*Gamma[1 + n, 3*ArcSinh[a*x]])/(8*a^3)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{arsinh}\,(ax)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsinh(a*x)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{arsinh}\,(ax)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^n, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{arcsinh}\,(ax)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^n,x)
```

```
[Out] int(x^2*arcsinh(a*x)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{arsinh}\,(ax)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asinh}(a\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x)^n,x)
```

```
[Out] int(x^2*asinh(a*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asinh}^n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**n,x)
```

```
[Out] Integral(x**2*asinh(a*x)**n, x)
```

3.132 $\int x \sinh^{-1}(ax)^n \, dx$

Optimal. Leaf size=59

$$\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1,-2\sinh^{-1}(ax)\right)}{a^2}+\frac{2^{-n-3}\Gamma\left(n+1,2\sinh^{-1}(ax)\right)}{a^2}$$

```
[Out] 2^(-3-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcsinh(a*x))/a^2/((-arcsinh(a*x))^n)+
2^(-3-n)*GAMMA(1+n,2*arcsinh(a*x))/a^2
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5669, 5448, 12, 3308, 2181}

$$\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-2\sinh^{-1}(ax)\right)}{a^2}+\frac{2^{-n-3}\text{Gamma}\left(n+1,2\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^2*(-ArcSinh[a*
x])^n) + (2^(-3 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x\sinh^{-1}(ax)^n\,dx = \frac{\text{Subst}\left(\int x^n\cosh(x)\sinh(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{2}x^n\sinh(2x)\,dx, x, \sinh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^n\sinh(2x)\,dx, x, \sinh^{-1}(ax)\right)}{2a^2}$$
$$= -\frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int e^{2x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{4a^2}$$
$$= \frac{2^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n, -2\sinh^{-1}(ax)\right)}{a^2} + \frac{2^{-3-n}\Gamma\left(1+n, 2\sinh^{-1}(\right.}{a^2}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.00

$$\frac{2^{-n-3}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\Gamma\left(n+1, -2\sinh^{-1}(ax)\right)}{a^2} + \frac{2^{-n-3}\Gamma\left(n+1, 2\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x]^n,x]
[Out] (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^2*(-ArcSinh[a*
x])^n) + (2^(-3 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^2
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{arsinh}\,(ax)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="fricas")
[Out] integral(x*arcsinh(a*x)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{arsinh}\,(ax)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="giac")
[Out] integrate(x*arcsinh(a*x)^n, x)
```

maple [C] time = 0.16, size = 38, normalized size = 0.64

$$\frac{\text{arcsinh}\,(ax)^{n+2}\,\text{hypergeom}\left(\left[\frac{n}{2}+1\right], \left[\frac{3}{2}, 2+\frac{n}{2}\right], \text{arcsinh}\,(ax)^2\right)}{a^2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^n,x)
[Out] 1/a^2/(n+2)*arcsinh(a*x)^(n+2)*hypergeom([1/2*n+1],[3/2,2+1/2*n],arcsinh(a*
x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arsinh}(ax)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{asinh}(a\,x)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x)^n,x)
```

```
[Out] int(x*asinh(a*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asinh}^n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**n,x)
```

```
[Out] Integral(x*asinh(a*x)**n, x)
```

3.133 $\int \sinh^{-1}(ax)^n\,dx$

Optimal. Leaf size=49

$$\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(n+1,-\sinh^{-1}(ax)\right)}{2a}-\frac{\Gamma\left(n+1,\sinh^{-1}(ax)\right)}{2a}$$

```
[Out] 1/2*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a/((-arcsinh(a*x))^n)-1/2*GAMMA
(1+n,arcsinh(a*x))/a
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5657, 3307, 2181}

$$\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)}{2a}-\frac{\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^n,x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a*(-ArcSinh[a*x])^n) - Gamm
a[1 + n, ArcSinh[a*x]]/(2*a)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^{-1}(ax)^n\,dx &= \frac{\text{Subst}\left(\int x^n\cosh(x)\,dx,x,\sinh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Subst}\left(\int e^{-x}x^n\,dx,x,\sinh^{-1}(ax)\right)}{2a}+\frac{\text{Subst}\left(\int e^{x}x^n\,dx,x,\sinh^{-1}(ax)\right)}{2a}\\ &= \frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-\sinh^{-1}(ax)\right)}{2a}-\frac{\Gamma\left(1+n,\sinh^{-1}(ax)\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.92

$$\frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(n+1,-\sinh^{-1}(ax)\right)-\Gamma\left(n+1,\sinh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^n,x]
```

```
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n - Gamma[1 +
 n, ArcSinh[a*x]])/(2*a)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\operatorname{arsinh}\left(ax\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}\left(ax\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n, x)
```

maple [C] time = 0.14, size = 40, normalized size = 0.82

$$\frac{\operatorname{arcsinh}\left(ax\right)^{n+1} \operatorname{hypergeom}\left(\left[\frac{n}{2}+\frac{1}{2}\right],\left[\frac{1}{2},\frac{n}{2}+\frac{3}{2}\right],\frac{\operatorname{arcsinh}(ax)^{2}}{4}\right)}{a\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n,x)
```

```
[Out] 1/a/(n+1)*arcsinh(a*x)^(n+1)*hypergeom([1/2*n+1/2],[1/2,1/2*n+3/2],1/4*arcs
inh(a*x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arsinh}\left(ax\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\mathrm{asinh}\left(a\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n,x)
```

```
[Out] int(asinh(a*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}^{n}{\left(ax \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n,x)
```

```
[Out] Integral(asinh(a*x)**n, x)
```

3.134 $\int \frac{\sinh^{-1}(ax)^n}{x} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\sinh^{-1}(ax)^n}{x},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/x,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/x, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x} \, dx = \int \frac{\sinh^{-1}(ax)^n}{x} \, dx$$

Mathematica [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^n}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/x,x]
```

```
[Out] Integrate[ArcSinh[a*x]^n/x, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^n/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^n}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n/x, x)
```

maple [A]   time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x,x)
```

```
[Out] int(arcsinh(a*x)^n/x,x)
```

maxima [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/x, x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\operatorname{asinh}(a\,x)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n/x,x)
```

```
[Out] int(asinh(a*x)^n/x, x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^n(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x,x)
```

```
[Out] Integral(asinh(a*x)**n/x, x)
```

3.135 $\int \frac{\sinh^{-1}(ax)^n}{x^2} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\sinh^{-1}(ax)^n}{x^2},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/x^2,x]
[Out] Defer[Int][ArcSinh[a*x]^n/x^2, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x^2} \, dx = \int \frac{\sinh^{-1}(ax)^n}{x^2} \, dx$$

Mathematica [A] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{-1}(ax)^n}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/x^2,x]
[Out] Integrate[ArcSinh[a*x]^n/x^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^n}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^n/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arsinh}\,(ax)^n}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^n/x^2, x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x^2,x)
```

```
[Out] int(arcsinh(a*x)^n/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arsinh}(ax)^n}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\operatorname{asinh}(a\, x)^n}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x)^n/x^2,x)
```

```
[Out] int(asinh(a*x)^n/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^n(ax)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x**2,x)
```

```
[Out] Integral(asinh(a*x)**n/x**2, x)
```

3.136 $\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-\frac{3a}{b}}$$

```
[Out] 1/144*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1
/2)*Pi^(1/2)/c^3-1/144*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/
2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-1/16*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3+1/16*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)+1/3*x^3*(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5663, 5779, 3312, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (x^3*Sqrt[a + b*ArcSinh[c*x]])/3 - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(48*c^3) + (Sqrt[b]*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx=\frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{1}{6}(bc)\int\frac{x^3}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

$$=\frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{b\,\text{Subst}\left(\int\frac{\sinh^3(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{6c^3}$$

$$=\frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{(ib)\,\text{Subst}\left(\int\left(\frac{3i\sinh(x)}{4\sqrt{a+bx}}-\frac{i\sinh(3x)}{4\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(cx)\right)}{6c^3}$$

$$=\frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{b\,\text{Subst}\left(\int\frac{\sinh(3x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{24c^3}+\frac{b\,\text{Subst}\left(\int\frac{\sinh}{\sqrt{a+}}\right.}{}$$

$$=\frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{b\,\text{Subst}\left(\int\frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{48c^3}-\frac{b\,\text{Subst}\left(\int\frac{e^{3x}}{\sqrt{a+b}}\right.}{}$$

$$=\frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{24c^3}-\frac{\text{Subst}\left(\right.}{}$$

$$=\frac{1}{3}x^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{b}\,e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a}}{}\right.}{48c^3}$$

Mathematica [A] time = 0.42, size = 215, normalized size = 1.01

$$\frac{e^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{3\left(a+b\sinh^{-1}(cx\right.}{b}\right.\right.}{72c^3\sqrt{-\frac{\left(a+b\sinh^{-1}(c\right.}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Ga
mma[3/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2,
```

```
(-3*(a + b*ArcSinh[c*x]))/b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[3/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSin
h[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sq
rt[-((a + b*ArcSinh[c*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(cx) + a}\, x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \operatorname{arcsinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(cx) + a}\, x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{a + b \operatorname{asinh}(c\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \operatorname{asinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*asinh(c*x)), x)
```

3.137 $\int x\sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}+\frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}$$

```
[Out] -1/32*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*2^(1
/2)*Pi^(1/2)/c^2-1/32*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2
)*2^(1/2)*Pi^(1/2)/c^2/exp(2*a/b)+1/4*(a+b*arcsinh(c*x))^(1/2)/c^2+1/2*x^2*
(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}+\frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] Sqrt[a + b*ArcSinh[c*x]]/(4*c^2) + (x^2*Sqrt[a + b*ArcSinh[c*x]])/2 - (Sqrt
[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/(16*c^2) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(16*c^2*E^((2*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{a+b\sinh^{-1}(cx)}\,dx &= \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{1}{4}(bc)\int\frac{x^2}{\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}\,dx\\
&= \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{b\,\mathrm{Subst}\left(\int\frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{4c^2}\\
&= \frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}+\frac{b\,\mathrm{Subst}\left(\int\left(\frac{1}{2\sqrt{a+bx}}-\frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(cx)\right)}{4c^2}\\
&= \frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{b\,\mathrm{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{8c^2}\\
&= \frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{b\,\mathrm{Subst}\left(\int\frac{e^{-2x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{16c^2}\\
&= \frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{\mathrm{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{8c^2}\\
&= \frac{\sqrt{a+b\sinh^{-1}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{\sqrt{b}\,e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^2}-
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 127, normalized size = 0.88

$$
\frac{e^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)}{8\sqrt{2}\,c^2\sqrt{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-2*(a + b*A
rcSinh[c*x]))/b] + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (
2*(a + b*ArcSinh[c*x]))/b]))/(8*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[-((a + b*ArcSi
nh[c*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(cx) + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int x\sqrt{a + b \operatorname{arcsinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arsinh}(cx) + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\, \sqrt{a + b \operatorname{asinh}(c\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a + b \operatorname{asinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*asinh(c*x)), x)
```

3.138 $\int \sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$\frac{\sqrt{\pi}\sqrt{b}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/4*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+x*(a+b*ar
csinh(c*x))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(cx)}\,dx = x\sqrt{a+b\sinh^{-1}(cx)} - \frac{1}{2}(bc)\int \frac{x}{\sqrt{1+c^2x^2}\,\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

$$= x\sqrt{a+b\sinh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2c}$$

$$= x\sqrt{a+b\sinh^{-1}(cx)} + \frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c} - \frac{b\,\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx\right.}{4c}$$

$$= x\sqrt{a+b\sinh^{-1}(cx)} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2c} - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\right.}{}$$

$$= x\sqrt{a+b\sinh^{-1}(cx)} + \frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{4c}$$

Mathematica [A] time = 0.24, size = 101, normalized size = 0.99

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}}-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/S
qrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((a
+ b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{arsinh}\,(cx)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\text{arcsinh}\,(cx)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2),x)
[Out] int((a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{arsinh}\,(cx) + a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\,\text{asinh}\,(c\,x)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(1/2),x)
[Out] int((a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\text{asinh}\,(cx)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2),x)
[Out] Integral(sqrt(a + b*asinh(c*x)), x)
```

3.139 $\int x^2 \left(a + b\sinh^{-1}(cx)\right)^{3/2} dx$

Optimal. Leaf size=282

$$-\frac{3\sqrt{\pi}\, b^{3/2}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}\, b^{3/2}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}\, b^{3/2}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}\, b}{}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))^(3/2)+1/288*b^(3/2)*exp(3*a/b)*erf(3^(1/2)*(a+b*
arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3+1/288*b^(3/2)*erfi(3^(1/2
)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-3/32*b^
(3/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3-3/32*b^(3
/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp(a/b)+1/3*b*(c^2
*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c^3-1/6*b*x^2*(c^2*x^2+1)^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/c
```

Rubi [A] time = 0.86, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5663, 5758, 5717, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{3\sqrt{\pi}\, b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}\, b^{3/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}\, b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*x^2*Sqrt[1 + c^
2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(6*c) + (x^3*(a + b*ArcSinh[c*x])^(3/2))/3
 - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c
^3) + (b^(3/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(96*c^3) - (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sq
rt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*Arc
Sinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}dx = \frac{1}{3}x^3\left(a+b\sinh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(bc)\int \frac{x^3\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}}dx$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{1}{12}b^2\int \ldots$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a \ldots\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a \ldots\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a \ldots\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a \ldots\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a \ldots\right.$$

$$= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + \frac{1}{3}x^3\left(a \ldots\right.$$

Mathematica [A] time = 0.31, size = 215, normalized size = 0.76

$$-\frac{be^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\Gamma\left(\frac{5}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\Gamma\left(\frac{5}{2},-\frac{3(a+b\sinh \ldots}{b}\right.\right.}{216c^3\sqrt{-\frac{(a+b\sinh \ldots}{\ldots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -1/216*(b*Sqrt[a + b*ArcSinh[c*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c
*x])/b)]*Gamma[5/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*
Gamma[5/2, (-3*(a + b*ArcSinh[c*x]))/b] - 27*E^((2*a)/b)*Sqrt[a/b + ArcSinh
[c*x]]*Gamma[5/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((
a + b*ArcSinh[c*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c*x]))/b]))/(c^3*E^((3
*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{arcsinh}(cx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{asinh}(c\, x)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asinh}(cx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**(3/2), x)
```

3.140 $\int x\left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=179

$$-\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}-\frac{3bx\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{8c}-$$

```
[Out] 1/4*(a+b*arcsinh(c*x))^(3/2)/c^2+1/2*x^2*(a+b*arcsinh(c*x))^(3/2)-3/128*b^(
3/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1
/2)/c^2+3/128*b^(3/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2
)*Pi^(1/2)/c^2/exp(2*a/b)-3/8*b*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2
)/c
```

Rubi [A] time = 0.48, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5663, 5758, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^2}-\frac{3bx\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*x*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(8*c) + (a + b*ArcSinh[
c*x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcSinh[c*x])^(3/2))/2 - (3*b^(3/2)*E^((2
*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*c^2)
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*c^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(cx)\right)^{3/2}dx=\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\frac{1}{4}(3bc)\int\frac{x^2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}}dx$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{1}{16}\left(3b^2\right)\int$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh\right.$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh\right.$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh\right.$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh\right.$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh\right.$$

$$=-\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{8c}+\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{4c^2}+\frac{1}{2}x^2\left(a+b\sinh\right.$$

Mathematica [A] time = 0.11, size = 129, normalized size = 0.72

$$\frac{be^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{5}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)}{16\sqrt{2}c^2\sqrt{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (b*Sqrt[a + b*ArcSinh[c*x]]*(-(Sqrt[a/b + ArcSinh[c*x]]*Gamma[5/2, (-2*(a +
 b*ArcSinh[c*x]))/b]) + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[5
/2, (2*(a + b*ArcSinh[c*x]))/b]))/(16*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[-((a + b
*ArcSinh[c*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\operatorname{arsinh}(cx)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**(3/2), x)
```

3.141 $\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=135

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\right.$$

```
[Out] x*(a+b*arcsinh(c*x))^(3/2)+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2
)/b^(1/2))*Pi^(1/2)/c+3/8*b^(3/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c/exp(a/b)-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

Rubi [A] time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^{3/2} dx = x\left(a+b\sinh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{1}{4}\left(3b^2\right)\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}} \cdots$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\cdots\right)}{\sqrt{x}} \cdots\right)}{\cdots}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\cdots\right)}}{\sqrt{x}} \cdots\right)}{\cdots}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{(3b)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\, d\cdots\right)}{\cdots}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + x\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\cdots}}{\cdots}\right)}{8c}$$

Mathematica [A] time = 1.11, size = 251, normalized size = 1.86

$$\frac{\sqrt{b}\left(4\sqrt{b}\left(2cx\sinh^{-1}(cx)-3\sqrt{c^2x^2+1}\right)\sqrt{a+b\sinh^{-1}(cx)}+\sqrt{\pi}(3b-2a)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}\cdots}}{\sqrt{b}}\right)\cdots\right)}{8c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*Ar
cSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a
+ 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
])))/(8*c)
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

maple [**F**] time = 0.39, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsinh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c\,x))^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int((a + b*asinh(c*x))^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(3/2), x)
```

3.142 $\int x^2 \left(a+b\sinh^{-1}(cx)\right)^{5/2} dx$

Optimal. Leaf size=327

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}+\frac{5\sqrt{\frac{\pi}{3}}\,b^{5/2}e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{576c^3}+\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}-\frac{5\sqrt{\frac{\pi}{3}}}{}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))^(5/2)+5/1728*b^(5/2)*exp(3*a/b)*erf(3^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3-5/1728*b^(5/2)*erfi(3^(1
/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-15/64
*b^(5/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3+15/64*
b^(5/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp(a/b)+5/9*b*
(a+b*arcsinh(c*x))^(3/2)*(c^2*x^2+1)^(1/2)/c^3-5/18*b*x^2*(a+b*arcsinh(c*x)
)^(3/2)*(c^2*x^2+1)^(1/2)/c-5/6*b^2*x*(a+b*arcsinh(c*x))^(1/2)/c^2+5/36*b^2
*x^3*(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5663, 5758, 5717, 5653, 5779, 3308, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}+\frac{5\sqrt{\frac{\pi}{3}}\,b^{5/2}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{576c^3}+\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^3}-\frac{5\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-5*b^2*x*Sqrt[a + b*ArcSinh[c*x]])/(6*c^2) + (5*b^2*x^3*Sqrt[a + b*ArcSinh
[c*x]])/36 + (5*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(9*c^3) - (
5*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(18*c) + (x^3*(a + b*
ArcSinh[c*x])^(5/2))/3 - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSin
h[c*x]]/Sqrt[b]])/(64*c^3) + (5*b^(5/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(576*c^3) + (15*b^(5/2)*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*c^3*E^(a/b)) - (5*b^(5/2)*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(576*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int x^2 \left(a+b\sinh^{-1}(cx)\right)^{5/2} dx = \frac{1}{3}x^3\left(a+b\sinh^{-1}(cx)\right)^{5/2} - \frac{1}{6}(5bc)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\sqrt{1+c^2x^2}}dx$$

$$= -\frac{5bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{18c} + \frac{1}{3}x^3\left(a+b\sinh^{-1}(cx)\right)^{5/2} + \frac{1}{12}\left(5b^2\right)\int$$

$$= \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1+c^2x}}{}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{9c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\sinh^{-1}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\sinh^{-1}(cx)} + \frac{5b\sqrt{1+c^2x^2}\left(a+b\sin\right.}{9c^3}$$

Mathematica [A] time = 0.50, size = 215, normalized size = 0.66

$$-\frac{e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(cx)\right)^{5/2}\left(81e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{7}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{7}{2},-\frac{3\left(a+b\sinh^{-1}(\right.}{b}\right.\right.}{648c^3\left(-\frac{\left(a+b\sinh^{-1}(cx\right.}{b^2}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] -1/648*((a + b*ArcSinh[c*x])^(5/2)*(81*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*
x])/b)]*Gamma[7/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*G
amma[7/2, (-3*(a + b*ArcSinh[c*x]))/b] - 81*E^((2*a)/b)*Sqrt[a/b + ArcSinh[
c*x]]*Gamma[7/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a
 + b*ArcSinh[c*x])/b)]*Gamma[7/2, (3*(a + b*ArcSinh[c*x]))/b]))/(c^3*E^((3*
a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} (a + b \operatorname{arcsinh}(cx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{2} (a + b \operatorname{asinh}(c\,x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c*x))^(5/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} (a + b \operatorname{asinh}(cx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**(5/2), x)
```

3.143 $\int x \left(a+b\sinh^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=223

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}+\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{}$$

```
[Out] 1/4*(a+b*arcsinh(c*x))^(5/2)/c^2+1/2*x^2*(a+b*arcsinh(c*x))^(5/2)-15/512*b^
(5/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/c^2-15/512*b^(5/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/c^2/exp(2*a/b)-5/8*b*x*(a+b*arcsinh(c*x))^(3/2)*(c^2*x^2+1)^(1
/2)/c+15/64*b^2*(a+b*arcsinh(c*x))^(1/2)/c^2+15/32*b^2*x^2*(a+b*arcsinh(c*x
))^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5663, 5758, 5675, 5779, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{256c^2}+\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (15*b^2*Sqrt[a + b*ArcSinh[c*x]])/(64*c^2) + (15*b^2*x^2*Sqrt[a + b*ArcSinh
[c*x]])/32 - (5*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(8*c) + (
a + b*ArcSinh[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcSinh[c*x])^(5/2))/2 - (1
5*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(256*c^2) - (15*b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(256*c^2*E^((2*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(cx)\right)^{5/2}dx=\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{5/2}-\frac{1}{4}(5bc)\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\sqrt{1+c^2x^2}}dx$$

$$=-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}+\frac{1}{2}x^2\left(a+b\sinh^{-1}(cx)\right)^{5/2}+\frac{1}{16}\left(15b^2\right)\int$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}+\frac{\left(a+b\sinh^{-1}\right.}{4c^2}$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}+\frac{\left(a+b\sinh^{-1}\right.}{4c^2}$$

$$=\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{8c}+\frac{\left(a+b\sinh^{-1}\right.}{4c^2}$$

$$=\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{8c}$$

$$=\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{8c}$$

$$=\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{8c}$$

$$=\frac{15b^2\sqrt{a+b\sinh^{-1}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\sinh^{-1}(cx)}-\frac{5bx\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{8c}$$

Mathematica [**A**] time = 0.08, size = 115, normalized size = 0.52

$$\frac{e^{-\frac{2a}{b}}\left(b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{7}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-b^3\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{7}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)}{32\sqrt{2}\,c^2\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-(b^3*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSinh[c*x]))
/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[7/2, (2*(a + b*ArcSin
h[c*x]))/b])/(32*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c*x))^(5/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**(5/2), x)
```

3.144 $\int \left(a+b\sinh^{-1}(cx)\right)^{5/2}\,dx$

Optimal. Leaf size=155

$$\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}+\frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{c^2x^2+1}\,(a$$

```
[Out] x*(a+b*arcsinh(c*x))^(5/2)+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1
/2)/b^(1/2))*Pi^(1/2)/c-15/16*b^(5/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c/exp(a/b)-5/2*b*(a+b*arcsinh(c*x))^(3/2)*(c^2*x^2+1)^(1/2)/c+15
/4*b^2*x*(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5653, 5717, 5779, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c}+\frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{c^2x^2+1}\,(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (15*b^2*x*Sqrt[a + b*ArcSinh[c*x]])/4 - (5*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x])^(3/2))/(2*c) + x*(a + b*ArcSinh[c*x])^(5/2) + (15*b^(5/2)*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c) - (15*b^(5/2)*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^{5/2}\,dx = x\left(a+b\sinh^{-1}(cx)\right)^{5/2}-\frac{1}{2}(5bc)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{5/2}+\frac{1}{4}\left(15b^2\right)\int\sqrt{a}$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\sinh^{-1}\right.$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\sinh^{-1}\right.$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\sinh^{-1}\right.$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\sinh^{-1}\right.$$

$$= \frac{15}{4}b^2x\sqrt{a+b\sinh^{-1}(cx)}-\frac{5b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{2c}+x\left(a+b\sinh^{-1}\right.$$

Mathematica [A] time = 3.65, size = 282, normalized size = 1.82

$$\sqrt{b}\,e^{-\frac{a}{b}}\left(-\left(\sqrt{\pi}\left(4a^2-15b^2\right)e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)\right)+\sqrt{\pi}\left(4a^2-15b^2\right)\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)+\frac{4\sqrt{b}\left(-2a^2e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (Sqrt[b]*(-((4*a^2 - 15*b^2)*E^((2*a)/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*
x]]/Sqrt[b]]) + (4*a^2 - 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqr
t[b]] + (4*Sqrt[b]*(E^(a/b)*(a + b*ArcSinh[c*x])*(5*(3*b*c*x - 2*a*Sqrt[1 +
 c^2*x^2]) + 2*(4*a*c*x - 5*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 4*b*c*x*Arc
```

```
Sinh[c*x]^2) - 2*a^2*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, a/b +
ArcSinh[c*x]] - 2*a^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, -((a + b*A
rcSinh[c*x])/b)]))/Sqrt[a + b*ArcSinh[c*x]]))/(16*c*E^(a/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsinh}(cx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c\,x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^(5/2),x)
```

```
[Out] int((a + b*asinh(c*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(cx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(5/2), x)
```

3.145 $\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=194

$$-\frac{\sqrt{\pi}\, e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\, e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\pi}\, e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\, e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}$$

```
[Out] 1/24*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/c^3/b^(1/2)+1/24*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)
*Pi^(1/2)/c^3/exp(3*a/b)/b^(1/2)-1/8*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/
b^(1/2))*Pi^(1/2)/c^3/b^(1/2)-1/8*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c^3/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5669, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\, e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\, e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\pi}\, e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\, e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3) +
 (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
8*Sqrt[b]*c^3) - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[
b]*c^3*E^(a/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}}+\frac{\cosh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^3}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3} - \frac{\text{Subst}\left(\int \cdots\right.}{}$$

$$= \frac{\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{4bc^3} - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right.}{4bc^3}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3} + \frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3} - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}\cdots}}{\sqrt{b}}\right.}{8\sqrt{b}\,c^3}$$

Mathematica [A] time = 0.23, size = 196, normalized size = 1.01

$$\frac{e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-3e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}\cdots}{b}}\right.}{24c^3\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sq
rt[3]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/
b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcS
inh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*
(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(c*x) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + b \operatorname{arcsinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a + b \operatorname{asinh}(c\, x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int(x^2/(a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + b \operatorname{asinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*asinh(c*x)), x)
```

3.146 $\int \frac{x}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=107

$$\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^2}-\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^2}$$

```
[Out] -1/8*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1
/2)/c^2/b^(1/2)+1/8*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/c^2/exp(2*a/b)/b^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^2}-\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -(E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
4*Sqrt[b]*c^2) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(4*Sqrt[b]*c^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2c^2}\\
&= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}+\frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}\\
&= -\frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2bc^2}+\frac{\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2bc^2}\\
&= -\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^2}+\frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^2}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 108, normalized size = 1.01

$$\frac{e^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)\right)}{4\sqrt{2}\,c^2\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*ArcSinh[c*x]],x]
[Out] (Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] +
E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b]
)/(4*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b*arcsinh(c*x) + a), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b \operatorname{arcsinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^(1/2),x)
[Out] int(x/(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a + b \operatorname{asinh}(c\, x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c*x))^(1/2),x)
[Out] int(x/(a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b \operatorname{asinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(x/sqrt(a + b*asinh(c*x)), x)
```

3.147 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{\pi}\,e^{a/b}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)+1/2*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5657, 3307, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{a/b}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc}$$
$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc}$$
$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 1.15

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2c\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) + S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*c*
E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\operatorname{arsinh}(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\operatorname{arcsinh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}(c\, x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{asinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c*x)), x)
```

3.148 $\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}\,e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b}\cdots}{\cdots}\right)}{4b^{3/2}c^3}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3-1/4
*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1/4*e
xp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^
(3/2)/c^3+1/4*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/
2)/b^(3/2)/c^3/exp(3*a/b)-2*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1
/2)
```

Rubi [A] time = 0.32, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5665, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}\,e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a+}\cdots}{\cdots}\right)}{4b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (E^((3*a)/b)*Sq
rt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) -
 (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b))
+ (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*
c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{2\,\text{Subst}\left(\int\left(-\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{3\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\right.}{2}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4bc^3} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\right.}{4b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2b^2c^3} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh}}{\sqrt{b}}\right.}{4b^{3/2}c^3}$$

Mathematica [A] time = 0.41, size = 290, normalized size = 1.28

$$\frac{e^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(e^{\frac{3a}{b}+2\sinh^{-1}(cx)}+e^{\frac{3a}{b}+4\sinh^{-1}(cx)}-e^{\frac{3a}{b}+6\sinh^{-1}(cx)}-e^{\frac{4a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{sinh}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c*x]) + E^((3*a)/b + 4*ArcSinh[c*x])
 - E^((3*a)/b + 6*ArcSinh[c*x]) - E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + A
rcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcSinh[c*x])*Sq
rt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - E^(
(2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a +
 b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3*E^(3*(a/b + ArcS
inh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a+b\,\mathrm{asinh}\,(c\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int(x^2/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\,\mathrm{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*asinh(c*x))**(3/2), x)
```

3.149 $\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

```
[Out] 1/2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/
2)/b^(3/2)/c^2+1/2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^2/exp(2*a/b)-2*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5665, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}-\frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^((2*a)/b)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*c^2) + (Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*c^2*E^(
(2*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2}+\frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx,\right.}{bc^2}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c^2}+\frac{2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}+\frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}}}{\sqrt{b}}\right.}{b^{3/2}c^2}$$

Mathematica [A] time = 0.11, size = 134, normalized size = 0.99

$$\frac{e^{-\frac{2a}{b}}\left(-2e^{\frac{2a}{b}}\sinh\left(2\sinh^{-1}(cx)\right)+\sqrt{2}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-\sqrt{2}\,e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b}{}\right.\right.}{2bc^2\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]
))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*A
rcSinh[c*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c*x]])/(2*b*c^2*E^((2*a)/b)
*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(x/(b*arcsinh(c*x) + a)^(3/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{arcsinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(x/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x/(b*arcsinh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c*x))^(3/2),x)
[Out] int(x/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**(3/2),x)
[Out] Integral(x/(a + b*asinh(c*x))**(3/2), x)
```

3.150 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a+
b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c^2*x^2+1)^(1
/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}+\frac{\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,\ldots\right.}{b\ldots}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c}+\frac{2\,\mathrm{Subst}\ldots}{\ldots}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c\ldots}}{\sqrt{b}}\right.}{b^{3/2}c}$$

Mathematica [A] time = 0.25, size = 137, normalized size = 1.18

$$\frac{e^{-\frac{a+b\sinh^{-1}(cx)}{b}}\left(-e^{a/b}\left(e^{2\sinh^{-1}(cx)}+1\right)+e^{\frac{2a}{b}+\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+e^{\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh\ldots}{\ldots}}\ldots\right.}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a + b*
ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{arsinh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\,\mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(3/2),x)
[Out] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{arsinh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\,\mathrm{asinh}\,(c\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^(3/2),x)
[Out] int(1/(a + b*asinh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\,\mathrm{asinh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**(-3/2), x)
```

3.151 $\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}\,e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}-\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}\,e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\ldots}{\ldots}\right.}{2b^{5/2}\ldots}$$

```
[Out] -1/6*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/c^3-1/
6*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/c^3/exp(a/b)+1/2*
exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b
^(5/2)/c^3+1/2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/b^(5/2)/c^3/exp(3*a/b)-2/3*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
^(3/2)-8/3*x/b^2/c^2/(a+b*arcsinh(c*x))^(1/2)-4*x^3/b^2/(a+b*arcsinh(c*x))^
(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5667, 5774, 5669, 5448, 3307, 2180, 2204, 2205, 5657}

$$-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}\,e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}-\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3}+\frac{\sqrt{3\pi}\,e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\ldots\right.}{2b^{5/\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - (8*x)/(3*b^
2*c^2*Sqrt[a + b*ArcSinh[c*x]]) - (4*x^3)/(b^2*Sqrt[a + b*ArcSinh[c*x]]) -
(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(6*b^(5/2)*c^3) +
(E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2
*b^(5/2)*c^3) - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(6*b^(5/2
)*c^3*E^(a/b)) + (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(2*b^(5/2)*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} + \frac{4\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}dx}{3bc} + \frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\right.}}{b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} +$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} +$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} +$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} +$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} +$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} +$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\sinh^{-1}(cx)}} -$$

Mathematica [A] time = 1.62, size = 340, normalized size = 1.25

$$e^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-6\sqrt{3}\,be^{3\sinh^{-1}(cx)}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+2be^{\frac{2a}{b}+3\sinh^{-1}(cx)}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (2*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x
])*Gamma[1/2, a/b + ArcSinh[c*x]] - 6*Sqrt[3]*b*E^(3*ArcSinh[c*x])*(-((a +
b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 2*b*E^(
(2*a)/b + 3*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a
 + b*ArcSinh[c*x])/b)] - E^((3*a)/b)*((-1 + E^(2*ArcSinh[c*x]))*(b*(-1 + E^
(4*ArcSinh[c*x])) + a*(6 + 4*E^(2*ArcSinh[c*x]) + 6*E^(4*ArcSinh[c*x])) + 2
*b*(3 + 2*E^(2*ArcSinh[c*x]) + 3*E^(4*ArcSinh[c*x]))*ArcSinh[c*x]) + 6*Sqrt
[3]*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x]
)*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(12*b^2*c^3*E^(3*(a/b + ArcSinh[
c*x]))*(a + b*ArcSinh[c*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{arcsinh}(cx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \operatorname{arsinh}(cx) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + b \operatorname{asinh}(c\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*asinh(c*x))^(5/2),x)
```

```
[Out] int(x^2/(a + b*asinh(c*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{asinh}(cx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x**2/(a + b*asinh(c*x))**(5/2), x)
```

3.152 $\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2\sqrt{2\pi}\,e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{2\sqrt{2\pi}\,e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}-\frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}}-\frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

```
[Out] -2/3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1
/2)/b^(5/2)/c^2+2/3*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(5/2)/c^2/exp(2*a/b)-2/3*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*
x))^(3/2)-4/3/b^2/c^2/(a+b*arcsinh(c*x))^(1/2)-8/3*x^2/b^2/(a+b*arcsinh(c*x
))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5667, 5774, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5675}

$$-\frac{2\sqrt{2\pi}\,e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}+\frac{2\sqrt{2\pi}\,e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}-\frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}}-\frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - 4/(3*b^2*c^2*
Sqrt[a + b*ArcSinh[c*x]]) - (8*x^2)/(3*b^2*Sqrt[a + b*ArcSinh[c*x]]) - (2*E
^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(3*b
^(5/2)*c^2) + (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(3*b^(5/2)*c^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} + \frac{2\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx}{3bc} + \frac{(4c)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\right.}}{3b}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\sinh^{-1}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}$$

Mathematica [A] time = 0.75, size = 200, normalized size = 1.09

$$\frac{e^{-2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(e^{\frac{2a}{b}}\left(4\sqrt{2}\,e^{2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\left(a+b\sinh^{-1}(cx)\right)\Gamma\left(\frac{1}{2},\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-4ae^{4\sinh^{-1}(\right.\right.}{6b^2c^2\left(a\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*b*E^(2*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/
2, (-2*(a + b*ArcSinh[c*x]))/b] + E^((2*a)/b)*(-4*a + b - 4*a*E^(4*ArcSinh[
c*x]) - b*E^(4*ArcSinh[c*x]) - 4*b*(1 + E^(4*ArcSinh[c*x]))*ArcSinh[c*x] +
4*Sqrt[2]*E^(2*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSin
h[c*x])*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b]))/(6*b^2*c^2*E^(2*(a/b + Arc
Sinh[c*x]))*(a + b*ArcSinh[c*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\operatorname{arsinh}(cx) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(5/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b\operatorname{arcsinh}(cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\operatorname{arsinh}(cx) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b\operatorname{asinh}(c\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c*x))^(5/2),x)
```

```
[Out] int(x/(a + b*asinh(c*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b\operatorname{asinh}(cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c*x))**(5/2), x)
```

3.153 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2\sqrt{c^2x^2+1}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}$$

```
[Out] 2/3*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/c+2/3*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/c/exp(a/b)-2/3*(c^2*
x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(3/2)-4/3*x/b^2/(a+b*arcsinh(c*x))^(1/2
)
```

Rubi [A] time = 0.27, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5655, 5774, 5657, 3307, 2180, 2205, 2204}

$$\frac{2\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2\sqrt{c^2x^2+1}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - (4*x)/(3*b^2*Sq
rt[a + b*ArcSinh[c*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(3*b^(5/2)*c) + (2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]
])/(3*b^(5/2)*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}+\frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}+\frac{4\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{3b^2}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}+\frac{4\,\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a\right.}{3b^3c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a\right.}{3b^3c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}+\frac{4\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+\right.}{3b^3c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{3bc\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}$$

Mathematica [A] time = 0.60, size = 181, normalized size = 1.27

$$\frac{e^{-\frac{a+b\sinh^{-1}(cx)}{b}}\left(-e^{a/b}\left(2a\left(e^{2\sinh^{-1}(cx)}-1\right)-2b\sinh^{-1}(cx)+be^{2\sinh^{-1}(cx)}\left(2\sinh^{-1}(cx)+1\right)+b\right)-2be^{\sinh^{-1}(cx)}\left(-\frac{a+}{}\right.\right.}{3b^2c\left(a+b\sinh\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c*x])) - 2*b*ArcSinh[c*x] + b*E^(2*A
rcSinh[c*x])*(1 + 2*ArcSinh[c*x]))) - 2*E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
 + ArcSinh[c*x]]*(a + b*ArcSinh[c*x])*Gamma[1/2, a/b + ArcSinh[c*x]] - 2*b*
E^ArcSinh[c*x]*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSin
h[c*x])/b)])/(3*b^2*c*E^((a + b*ArcSinh[c*x])/b)*(a + b*ArcSinh[c*x])^(3/2)
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-5/2), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(a+b\,\mathrm{asinh}\left(c\,x\right))}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^(5/2),x)
```

```
[Out] int(1/(a + b*asinh(c*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-5/2), x)
```

3.154 $\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=346

$$\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}-\frac{3\sqrt{3\pi}\,e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}-\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3\sqrt{3\pi}\,e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\ldots}{}\right)}{5b^{7/2}\ldots}$$

```
[Out] -8/15*x/b^2/c^2/(a+b*arcsinh(c*x))^(3/2)-4/5*x^3/b^2/(a+b*arcsinh(c*x))^(3/
2)+1/15*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/c^3
-1/15*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/c^3/exp(a/b)-
3/5*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/
2)/b^(7/2)/c^3+3/5*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*P
i^(1/2)/b^(7/2)/c^3/exp(3*a/b)-2/5*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c
*x))^(5/2)-16/15*(c^2*x^2+1)^(1/2)/b^3/c^3/(a+b*arcsinh(c*x))^(1/2)-24/5*x^
2*(c^2*x^2+1)^(1/2)/b^3/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5667, 5774, 5665, 3308, 2180, 2204, 2205, 5655, 5779}

$$\frac{\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}-\frac{3\sqrt{3\pi}\,e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}-\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3\sqrt{3\pi}\,e^{-\frac{3a}{b}}\operatorname{Erfi}\left(\ldots\right)}{5b^{7/\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - (8*x)/(15*b
^2*c^2*(a + b*ArcSinh[c*x])^(3/2)) - (4*x^3)/(5*b^2*(a + b*ArcSinh[c*x])^(3
/2)) - (16*Sqrt[1 + c^2*x^2])/(15*b^3*c^3*Sqrt[a + b*ArcSinh[c*x]]) - (24*x
^2*Sqrt[1 + c^2*x^2])/(5*b^3*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c^3) - (3*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(5*b^(7/2)*c^3)
 - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c^3*E^(a/b
)) + (3*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(5*b^(
7/2)*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx = -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} + \frac{4\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{5/2}}dx}{5bc} + \frac{(6c)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\right.}}{5b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{8x}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{4x^3}{5b^2\left(a+b\sinh^{-1}(c\right.}$$

Mathematica [A] time = 1.74, size = 417, normalized size = 1.21

$$\frac{e^{-\sinh^{-1}(cx)}\left(4a^2+2b(4a-b)\sinh^{-1}(cx)-4e^{\frac{a}{b}+\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\left(a+b\sinh^{-1}(cx)\right)^2\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (3*b^2*E^ArcSinh[c*x] + (4*a^2 - 2*a*b + 3*b^2 + 2*(4*a - b)*b*ArcSinh[c*x]
 + 4*b^2*ArcSinh[c*x]^2 - 4*E^(a/b + ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]
*(a + b*ArcSinh[c*x])^2*Gamma[1/2, a/b + ArcSinh[c*x]])/E^ArcSinh[c*x] - 3*
(b^2*E^(3*ArcSinh[c*x]) + (2*(a + b*ArcSinh[c*x])*(E^(3*(a/b + ArcSinh[c*x]
))*(6*a + b + 6*b*ArcSinh[c*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c*x])/b))^(
3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b]))/E^((3*a)/b)) + (2*(a + b*Arc
Sinh[c*x])*(E^(a/b + ArcSinh[c*x])*(2*a + b + 2*b*ArcSinh[c*x]) + 2*b*(-((a
 + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)]))/E^(a/b
) - (3*(b^2 + 2*(a + b*ArcSinh[c*x])*(6*a - b + 6*b*ArcSinh[c*x] - 6*Sqrt[3
]*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x])*
Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])))/E^(3*ArcSinh[c*x]))/(60*b^3*c^3*(
a + b*ArcSinh[c*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^(7/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*asinh(c*x))^(7/2),x)
```

```
[Out] int(x^2/(a + b*asinh(c*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\operatorname{asinh}(cx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**(7/2),x)
```

```
[Out] Integral(x**2/(a + b*asinh(c*x))**(7/2), x)
```

3.155 $\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=219

$$\frac{8\sqrt{2\pi}\,e^{\frac{2a}{b}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{8\sqrt{2\pi}\,e^{-\frac{2a}{b}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}-\frac{32x\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}-\frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}$$

```
[Out] -4/15/b^2/c^2/(a+b*arcsinh(c*x))^(3/2)-8/15*x^2/b^2/(a+b*arcsinh(c*x))^(3/2
)+8/15*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(7/2)/c^2+8/15*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(7/2)/c^2/exp(2*a/b)-2/5*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh
(c*x))^(5/2)-32/15*x*(c^2*x^2+1)^(1/2)/b^3/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5667, 5774, 5665, 3307, 2180, 2204, 2205, 5675}

$$\frac{8\sqrt{2\pi}\,e^{\frac{2a}{b}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{8\sqrt{2\pi}\,e^{-\frac{2a}{b}}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}-\frac{32x\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}-\frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - 4/(15*b^2*c^2
*(a + b*ArcSinh[c*x])^(3/2)) - (8*x^2)/(15*b^2*(a + b*ArcSinh[c*x])^(3/2))
- (32*x*Sqrt[1 + c^2*x^2])/(15*b^3*c*Sqrt[a + b*ArcSinh[c*x]]) + (8*E^((2*a
)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(15*b^(7/2
)*c^2) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(1
5*b^(7/2)*c^2*E^((2*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5667

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-
Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcSinh[c*x])^(n + 1))/
Sqrt[1 + c^2*x^2], x], x] - Dist[m/(b*c*(n + 1)), Int[(x^(m - 1)*(a + b*Arc
Sinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}dx = -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} + \frac{2\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{5/2}}dx}{5bc} + \frac{(4c)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\right.}}{5b}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(c\right.}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}} - \frac{4}{15b^2c^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}} - \frac{8x^2}{15b^2\left(a+b\sinh^{-1}(c\right.}$$

Mathematica [A] time = 1.22, size = 208, normalized size = 0.95

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)\left(e^{-\frac{2a}{b}}\left(2e^{2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(4a+4b\sinh^{-1}(cx)+b\right)+8\sqrt{2}\,b\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\sinh\right.}{b}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] -1/15*((a + b*ArcSinh[c*x])*((2*E^(2*(a/b + ArcSinh[c*x]))*(4*a + b + 4*b*A
rcSinh[c*x]) + 8*Sqrt[2]*b*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, (-2
*(a + b*ArcSinh[c*x]))/b])/E^((2*a)/b) + (-8*a + 2*b - 8*b*ArcSinh[c*x] + 8
*Sqrt[2]*E^(2*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh
[c*x])*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b])/E^(2*ArcSinh[c*x])) + 3*b^2*
Sinh[2*ArcSinh[c*x]])/(b^3*c^2*(a + b*ArcSinh[c*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\operatorname{arsinh}(cx)+a\right)^{\frac{7}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(7/2), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^(7/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(a+b\,\mathrm{asinh}\,(c\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c*x))^(7/2),x)
```

```
[Out] int(x/(a + b*asinh(c*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+b\,\mathrm{asinh}\,(cx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**(7/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c*x))**(7/2), x)
```

3.156 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{4\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}-\frac{8\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx\right.}$$

```
[Out] -4/15*x/b^2/(a+b*arcsinh(c*x))^(3/2)-4/15*exp(a/b)*erf((a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/c+4/15*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/b^(7/2)/c/exp(a/b)-2/5*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^
(5/2)-8/15*(c^2*x^2+1)^(1/2)/b^3/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5655, 5774, 5779, 3308, 2180, 2204, 2205}

$$-\frac{8\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - (4*x)/(15*b^2*(
a + b*ArcSinh[c*x])^(3/2)) - (8*Sqrt[1 + c^2*x^2])/(15*b^3*c*Sqrt[a + b*Arc
Sinh[c*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(1
5*b^(7/2)*c) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7
/2)*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{7/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}}+\frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^{5/2}}\,dx}{5b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}+\frac{4\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx}{15b^2}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{5bc\left(a+b\sinh^{-1}(cx)\right)^{5/2}}-\frac{4x}{15b^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}-\frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\sinh^{-1}(cx)}}$$

Mathematica [A] time = 0.63, size = 210, normalized size = 1.18

$$-2e^{-\sinh^{-1}(cx)}\left(4a^2+2ab\left(4\sinh^{-1}(cx)-1\right)+b^2\left(4\sinh^{-1}(cx)^2-2\sinh^{-1}(cx)+3\right)\right)+8e^{a/b}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\left(a+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c*x]) + b^2*(3 -
 2*ArcSinh[c*x] + 4*ArcSinh[c*x]^2)))/E^ArcSinh[c*x] + 8*E^(a/b)*Sqrt[a/b +
 ArcSinh[c*x]]*(a + b*ArcSinh[c*x])^2*Gamma[1/2, a/b + ArcSinh[c*x]] - (4*(
a + b*ArcSinh[c*x])*(E^(a/b + ArcSinh[c*x])*(2*a + b + 2*b*ArcSinh[c*x]) +
2*b*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)]
))/E^(a/b))/(30*b^3*c*(a + b*ArcSinh[c*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-7/2), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{arcsinh}(cx))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a + b \operatorname{asinh}(c\, x)\right)}^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(c*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\,\mathrm{asinh}\,(cx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-7/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```